Exercise walk-through

Defining Polar Coordinates and Differentiating in Polar Form ▪ 9.7

eXercise

You must be able to

- convert between **polar 极坐标** (r, θ) and Cartesian (x, y)
- treat $r = f(\theta)$ as a parametric curve $x = r\cos\theta$, $y = r\sin\theta$
- find $\dfrac{dy}{dx}$ for a polar curve

Method — Polar to Cartesian

A point (r, θ) has $x = r\cos\theta$, $y = r\sin\theta$. So $(2, \frac{\pi}{3})$ is $x = 2\cos\frac{\pi}{3} = 1$, $y = 2\sin\frac{\pi}{3} = \sqrt{3}$.

Method — A polar curve is parametric in

For $r = f(\theta)$, use θ as the parameter:

$$x = f(\theta)\cos\theta, \qquad y = f(\theta)\sin\theta.$$

Method — Slope of a polar curve

The slope is the parametric slope with θ as parameter:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}.$$

Compute $\dfrac{dx}{d\theta}$ and $\dfrac{dy}{d\theta}$ from the product rule.

Method — A worked conversion

The circle $r = 4$ is $x = 4\cos\theta$, $y = 4\sin\theta$ — the circle of radius 4. The line $\theta = \frac{\pi}{4}$ is $y = x$ (for $r > 0$).

Practice 2.1 ▪ 2 marks

Convert the polar point $(3, \frac{\pi}{2})$ to Cartesian coordinates.

Solution 2.1

Method: Polar to Cartesian

Worked steps

$x = 3\cos\frac{\pi}{2} = 0,\ y = 3\sin\frac{\pi}{2} = 3$: $(0,3)$ M1 A1.

Practice 2.2 ▪ 2 marks

Write x and y for the polar curve $r = 2\cos\theta$.

Solution 2.2

Worked steps

$x = 2\cos\theta\cos\theta = 2\cos^2\theta,\ y = 2\cos\theta\sin\theta$ M1 A1.

Practice 2.3 ▪ 2 marks

Convert the Cartesian point $(0, 5)$ to polar coordinates.

Solution 2.3

Method: Polar to Cartesian

Worked steps

$r = 5$, $\theta = \frac{\pi}{2}$: $(5, \frac{\pi}{2})$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

For a polar curve $r = f(\theta)$, $x =$

A $r\sin\theta$

B $r\cos\theta$

C $\dfrac{r}{\cos\theta}$

D $r + \cos\theta$

Solution 3.1

A $r \sin\theta$

B $r \cos\theta$ ✓

C $\dfrac{r}{\cos\theta}$

D $r + \cos\theta$

Worked steps

B — $x = r\cos\theta$.

Exam-style 3.2 ▪ 2 marks

For the polar curve $r = 1 + \cos\theta$:

(a) Write x and y as functions of θ.

(b) Find the point (in Cartesian coordinates) at $\theta = 0$.

Solution 3.2

Method: Polar to Cartesian

Worked steps

(a) $x = (1 + \cos\theta)\cos\theta$, $y = (1 + \cos\theta)\sin\theta$ **M1** **A1**. (b) at $\theta = 0$: $r = 2$, $x = 2$, $y = 0$: $(2, 0)$ **M1** **A1**.

Exam-style 3.3 ▪ 3 marks

For $r = 2\sin\theta$, find $\dfrac{dx}{d\theta}$.

Solution 3.3

Method: Slope of a polar curve

Worked steps

$x = 2\sin\theta\cos\theta = \sin(2\theta)$ M1; $\dfrac{dx}{d\theta} = 2\cos(2\theta)$ M1 A1. (Or via product rule on $2\sin\theta\cos\theta$.)