# Exercise walk-through

## Integrating Vector-Valued Functions ▪ 9.5

eXercise

# You must be able to

- integrate a vector function **component by component**
- use an **initial condition** to find each constant
- recover velocity from acceleration, or position from velocity

# Method — Integrate each component

$\int \vec{r}(t)\, dt$ integrates each component separately, giving a **vector** of constants $\vec{C}$:

$$\int \langle f(t), g(t) \rangle \, dt = \left\langle \int f\, dt, \int g\, dt \right\rangle + \vec{C}.$$

# Method — From acceleration to velocity

Given $\vec{a}(t) = \langle 2, -6t \rangle$ and $\vec{v}(0) = \langle 1, 0 \rangle$: integrate, $\vec{v}(t) = \langle 2t + C_1, -3t^2 + C_2 \rangle$. Apply $\vec{v}(0) = \langle 1, 0 \rangle$: $C_1 = 1$, $C_2 = 0$, so $\vec{v}(t) = \langle 2t + 1, -3t^2 \rangle$.

# Method — From velocity to position

Integrate $\vec{v}$ and use $\vec{r}(0)$ to fix the constants. Each component is a separate initial-value problem.

# Method — Definite integral of a vector

$\int_a^b \vec{v}(t)\, dt$ gives the **displacement** vector over $[a, b]$ — integrate each component between the limits.

# Practice 2.1 ▪ 2 marks

Find $\displaystyle\int \langle 2t,\, 3\rangle \, dt$ (include the constant vector).

# Solution 2.1

Method: Integrate each component

## Worked steps

$\langle t^2, 3t\rangle + \vec{C}$ **M1** **A1**.

# Practice 2.2 ▪ 3 marks

Given $\vec{a}(t) = \langle 0, -10 \rangle$ and $\vec{v}(0) = \langle 5, 0 \rangle$, find $\vec{v}(t)$.

# Solution 2.2

Method: From acceleration to velocity

## Worked steps

$\vec{v}(t) = \langle C_1, -10t + C_2\rangle$; $\vec{v}(0) = \langle 5, 0\rangle \Rightarrow C_1 = 5, C_2 = 0$; $\vec{v}(t) = \langle 5, -10t\rangle$ **M1** **M1** **A1**.

# Practice 2.3 ▪ 2 marks

Evaluate $\int_0^2 \langle 1,\ 2t \rangle \, dt$.

# Solution 2.3

Method: Integrate each component

## Worked steps

$\left\langle [t]_0^2, [t^2]_0^2 \right\rangle = \langle 2, 4 \rangle$ **M1** **A1**.

# Exam-style 3.1 ▪ 1 mark

To integrate a vector-valued function you

**A** integrate each component separately

**B** take the magnitude first

**C** differentiate it

**D** add the components then integrate

# Solution 3.1

**A** integrate each component separately ✓

**B** take the magnitude first

**C** differentiate it

**D** add the components then integrate

**Worked steps**

**A** — integrate each component separately.

# Exam-style 3.2 ▪ 3 marks

A particle has acceleration $\vec{a}(t) = \langle 6t, 2 \rangle$, with $\vec{v}(0) = \langle 0, 1 \rangle$.
(a) Find $\vec{v}(t)$.
(b) Find $\vec{v}(1)$.

# Solution 3.2

Method: From acceleration to velocity

## Worked steps

(a) $\vec{v}(t) = \langle 3t^2 + C_1, 2t + C_2 \rangle$; $\vec{v}(0) = \langle 0, 1 \rangle \Rightarrow C_1 = 0, C_2 = 1$; $\vec{v}(t) = \langle 3t^2, 2t + 1 \rangle$ **M1** **M1** **A1**. (b) $\vec{v}(1) = \langle 3, 3 \rangle$ **B1**.

# Exam-style 3.3 ▪ 4 marks

A particle has velocity $\vec{v}(t) = \langle 2t, \cos t \rangle$ and starts at $\vec{r}(0) = \langle 1, 0 \rangle$. Find the position $\vec{r}(t)$.

# Solution 3.3

Method: From acceleration to velocity

## Worked steps

$\vec{r}(t) = \langle t^2 + C_1, \sin t + C_2 \rangle$ **M1**; $\vec{r}(0) = \langle 1, 0 \rangle \Rightarrow C_1 = 1, C_2 = 0$ **M1** **M1**; $\vec{r}(t) = \langle t^2 + 1, \sin t \rangle$ **A1**.