Exercise walk-through

Defining and Differentiating Parametric Equations ▪ 9.1

eXercise

You must be able to

- describe a curve by **parametric equations 参数方程** $x = x(t)$, $y = y(t)$
- find the slope $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$
- evaluate the slope at a given t

Method — Parametric curves

A parametric curve gives x and y each as a function of a parameter t: as t varies, the point $(x(t), y(t))$ traces the curve. Example: $x = t^2$, $y = t^3$.

Method — The slope formula

The chain rule gives the slope of the curve:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \qquad (dx/dt \neq 0).$$

Method — A worked slope

For $x = t^2$, $y = t^3$: $\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2$, so

$$\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}.$$

At $t = 2$ the slope is 3.

Method — Horizontal and vertical tangents

A **horizontal** tangent needs $\dfrac{dy}{dt} = 0$ (and $\dfrac{dx}{dt} \neq 0$); a **vertical** tangent needs $\dfrac{dx}{dt} = 0$.

Practice 2.1 ▪ 2 marks

For $x = t^2$, $y = t^3$, state $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$.

Solution 2.1

Worked steps

$$\frac{dx}{dt} = 2t,\ \frac{dy}{dt} = 3t^2$$ **B1** **B1**.

Practice 2.2 ▪ 2 marks

For $x = \cos t,\ y = \sin t$, find $\dfrac{dy}{dx}$.

Solution 2.2

Worked steps

$$\frac{dy}{dx} = \frac{\cos t}{-\sin t} = -\cot t \quad \boxed{\textbf{M1}}\boxed{\textbf{A1}}.$$

Practice 2.3 ▪ 2 marks

For $x = t^2$, $y = 4t$, find the slope at $t = 1$.

Solution 2.3

Worked steps

$\dfrac{dy}{dx} = \dfrac{4}{2t} = \dfrac{2}{t}$; at $t = 1$ slope $= 2$ M1 A1.

Exam-style 3.1 ▪ 1 mark

For a parametric curve, $\dfrac{dy}{dx} =$

A $\dfrac{dx/dt}{dy/dt}$

B $\dfrac{dy/dt}{dx/dt}$

C $\dfrac{dy}{dt} \cdot \dfrac{dx}{dt}$

D $\dfrac{d^2y}{dt^2}$

Solution 3.1

Method: Parametric curves

A $\dfrac{dx/dt}{dy/dt}$

B $\dfrac{dy/dt}{dx/dt}$ ✓

C $\dfrac{dy}{dt} \cdot \dfrac{dx}{dt}$

D $\dfrac{d^2y}{dt^2}$

Worked steps

B $= \dfrac{dy}{} = \dfrac{dy/dt}{}$

Exam-style 3.2 ▪ 2 marks

A curve is given by $x = t^2 - 1$, $y = t^3 - 3t$.

(a) Find $\dfrac{dy}{dx}$ in terms of t.

(b) Find the value(s) of t where the tangent is horizontal.

Solution 3.2

Method: Horizontal and vertical tangents

Worked steps

(a) $\dfrac{dy}{dx} = \dfrac{3t^2 - 3}{2t}$ M1 A1. (b) horizontal where $3t^2 - 3 = 0 \Rightarrow t = \pm 1$ M1 A1.

Exam-style 3.3 ▪ 3 marks

For $x = e^t$, $y = e^{2t}$, find the slope of the tangent line at $t = 0$.

Solution 3.3

Worked steps

$\dfrac{dx}{dt} = e^t$, $\dfrac{dy}{dt} = 2e^{2t}$ **M1**; $\dfrac{dy}{dx} = \dfrac{2e^{2t}}{e^t} = 2e^t$ **M1**; at $t = 0$, slope $= 2$ **A1**.