Exercise walk-through

Volume with Disc Method: Revolving Around the x- or y-Axis ▪ 8.9

eXercise

You must be able to

- revolve a region around an axis and integrate $\pi(\text{radius})^2$ (the **disc method 圆盘法**)
- set the radius as the distance from the curve to the axis
- integrate in x (around the x-axis) or y (around the y-axis)

Method — The disc method

Revolving a region under $y = f(x)$ around the x-axis makes discs of radius $f(x)$:

$$V = \pi \int_a^b \big(f(x)\big)^2 \, dx.$$

Method — A worked disc volume

Revolve $y = \sqrt{x}$, $0 \le x \le 4$, around the x-axis: radius $\sqrt{x}$, so

$$V = \pi \int_0^4 (\sqrt{x})^2 \, dx = \pi \int_0^4 x \, dx = \pi \cdot 8 = 8\pi.$$

Method — Around the y-axis

Revolving around the y-axis uses $x = f(y)$ and integrates in y: $V = \pi \int_c^d \big(x(y)\big)^2\, dy$.

For $y = x^2$ (so $x = \sqrt{y}$) from $y = 0$ to 4: $V = \pi \int_0^4 (\sqrt{y})^2\, dy = \pi \int_0^4 y\, dy = 8\pi$.

Method — Radius is a distance

The radius is always the **distance from the curve to the axis** of revolution — here just $f(x)$ or $x(y)$ because the axis is a coordinate axis.

Practice 2.1 ▪ 1 mark

Write the disc-method integral for revolving $y = x$ on $[0, 2]$ about the x-axis.

Solution 2.1

Method: The disc method

Worked steps

$\pi \int_0^2 x^2 \, dx$ **B1**.

Practice 2.2 ▪ 2 marks

Evaluate $\pi \int_0^2 x^2 \, dx$.

Solution 2.2

Worked steps

$\pi\left[\frac{x^3}{3}\right]_0^2 = \frac{8\pi}{3}$ M1 A1.

Practice 2.3 ▪ 3 marks

Find the volume when $y = \sqrt{x}$ on $[0, 9]$ is revolved about the x-axis.

Solution 2.3

Method: A worked disc volume

Worked steps

$$\pi \int_0^9 x\,dx = \pi \left[\frac{x^2}{2}\right]_0^9 = \frac{81\pi}{2}$$ M1 M1 A1.

Exam-style 3.1 ▪ 1 mark

Revolving $y = f(x)$ about the x-axis gives the volume

A $\displaystyle\int_a^b f(x)\,dx$

B $\displaystyle\pi\int_a^b f(x)\,dx$

C $\displaystyle\pi\int_a^b \big(f(x)\big)^2\,dx$

D $\displaystyle 2\pi\int_a^b f(x)\,dx$

Solution 3.1

A $\displaystyle\int_a^b f(x)\,dx$

B $\displaystyle\pi\int_a^b f(x)\,dx$

C $\displaystyle\pi\int_a^b \big(f(x)\big)^2\,dx$ ✓

D $\displaystyle 2\pi\int_a^b f(x)\,dx$

Worked steps

C — the disc method integrates $\pi(\text{radius})^2$.

Exam-style 3.2 ▪ 2 marks

The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the x-axis.
(a) Write the volume integral.
(b) Evaluate it.

Solution 3.2

Method: The disc method

Worked steps

(a) $\pi \int_0^2 (x^2)^2 \, dx = \pi \int_0^2 x^4 \, dx$ M1 A1. (b) $\pi \left[\frac{x^5}{5}\right]_0^2 = \frac{32\pi}{5}$ M1 M1 A1.

Exam-style 3.3 ▪ 4 marks

The region bounded by $x = y^2$, $y = 0$ to $y = 2$, and the y-axis is revolved about the y-axis. Find the volume.

Solution 3.3

Worked steps

radius $= x = y^2$; $V = \pi \int_0^2 (y^2)^2\, dy = \pi \int_0^2 y^4\, dy$ M1 M1 $= \pi \left[\frac{y^5}{5}\right]_0^2 = \frac{32\pi}{5}$ M1 A1.