Exercise walk-through

Using Accumulation Functions and Definite Integrals in Applied Contexts ▪ 8.3

eXercise

You must be able to

- interpret $\displaystyle\int_a^b R(t)\,dt$ as the accumulated amount from a rate R
- combine a starting amount with $\displaystyle\int(\text{rate in} - \text{rate out})$
- attach correct **units** and interpret in context

Method — Total from a rate

If a quantity changes at rate $R(t)$, the total change over $[a, b]$ is $\int_a^b R(t)\,dt$, carrying the units of $R\times$ time. Water flowing at $R(t) = 4t$ L/min adds $\int_0^3 4t\,dt = [2t^2]_0^3 = 18$ L over the first 3 minutes.

Method — Starting amount plus accumulation

The amount at time b is

$$\text{(start)} + \int_a^b R(t)\,dt.$$

A tank with 50 L that gains at $R(t) = 4t$ over $[0, 3]$ holds $50 + 18 = 68$ L at $t = 3$.

Method — Rate in minus rate out

If two rates act, integrate their **difference**. Water in at $I(t)$ and out at $O(t)$: net change $= \displaystyle\int_a^b \big(I(t) - O(t)\big)\, dt$.

Method — Interpreting a value

Always state what the number **means**: "$\int_0^3 R\,dt = 18$ litres of water were added in the first 3 minutes" — a number **with** units and meaning.

Practice 2.1 ▪ 1 mark

Water flows in at $R(t) = 6$ L/min for 10 minutes. How much enters?

Solution 2.1

Worked steps

$6 \times 10 = 60$ L **B1**.

Practice 2.2 ▪ 2 marks

A tank starts with 20 L and gains water at $R(t) = 2t$ L/min. Find the volume added over $0 \leq t \leq 4$.

Solution 2.2

Method: Starting amount plus accumulation

Worked steps

$$\int_0^4 2t\,dt = [t^2]_0^4 = 16 \text{ L}$$ **M1** **A1**.

Practice 2.3 ▪ 1 mark

For the tank in 2.2, find the total volume in the tank at $t = 4$.

Solution 2.3

Worked steps

$20 + 16 = 36$ L **B1**.

Exam-style 3.1 ▪ 1 mark

If $R(t)$ is a rate in litres per hour, then $\int_0^5 R(t)\,dt$ has units of

A litres per hour

B litres

C hours

D litres per hour squared

Solution 3.1

A litres per hour

B litres ✓

C hours

D litres per hour squared

Worked steps

B — (litres/hour) $\times$ hours $=$ litres.

Exam-style 3.2 ▪ 3 marks

A city uses electricity at a rate $P(t) = 40 + 10t$ megawatts for $0 \leq t \leq 6$ hours.
(a) Find the total energy used over the 6 hours.
(b) State the units.

Solution 3.2

Method: Total from a rate

Worked steps

(a) $\int_0^6 (40+10t)\,dt = \left[40t+5t^2\right]_0^6 = 240+180 = 420$ **M1** **M1** **A1**. (b) megawatt-hours **B1**.

Exam-style 3.3 ▪ 2 marks

A reservoir holds 500 m^3. Water enters at $I(t) = 30$ m^3/h and leaves at $O(t) = 6t$ m^3/h, for $0 \leq t \leq 5$.
(a) Write an expression for the amount of water at time $t = 5$.
(b) Evaluate it.

Solution 3.3

Method: Starting amount plus accumulation

Worked steps

(a) $500 + \int_0^5 (30 - 6t)\,dt$ **M1** **A1**. (b) $\left[30t - 3t^2\right]_0^5 = 150 - 75 = 75$ **M1** **M1**; total $500 + 75 = 575$ m^3 **A1**.