Exercise walk-through

Connecting Position, Velocity, and Acceleration of Functions Using Integrals ▪ 8.2

eXercise

You must be able to

- integrate acceleration to get velocity and velocity to get position
- find **displacement 位移** $\int_a^b v\,dt$ and **total distance 总路程** $\int_a^b |v|\,dt$
- use an initial condition to fix the constant

Method — Integrating the motion chain

Acceleration integrates to velocity; velocity integrates to position:

$$v(t) = \int a(t)\,dt, \qquad s(t) = \int v(t)\,dt.$$

Each integration adds a constant, fixed by an initial condition.

Method — Displacement

Displacement over $[a, b]$ is the net change in position:

$$\int_a^b v(t)\,dt = s(b) - s(a).$$

It can be negative (ended left of the start).

Method — Total distance

Total distance counts all motion regardless of direction:

$$\int_a^b |v(t)|\, dt.$$

Split the integral where v changes sign, integrate each piece, and add the **absolute** amounts.

Method — A worked distance

$v(t) = t - 2$ on $[0,3]$: $v < 0$ on $[0,2)$, $v > 0$ on $(2,3]$. Distance $= \left| \int_0^2 (t-2)\,dt \right| + \int_2^3 (t-2)\,dt = |-2| + \frac{1}{2} = 2.5.$

Practice 2.1 ▪ 2 marks

A particle has velocity $v(t) = 6t$. Find its displacement over $0 \leq t \leq 3$.

Solution 2.1

Method: Displacement

Worked steps

$$\int_0^3 6t\,dt = \left[3t^2\right]_0^3 = 27 \quad \text{M1}\ \text{A1}.$$

Practice 2.2 ▪ 2 marks

With $a(t) = 4$ and $v(0) = 1$, find $v(t)$.

Solution 2.2

Worked steps

$v(t) = 4t + C$; $v(0) = C = 1$, so $v(t) = 4t + 1$ **M1** **A1**.

Practice 2.3 ▪ 1 mark

A particle has $v(t) = 2t - 6$. State the time it changes direction.

Solution 2.3

Method: Total distance

Worked steps

$v = 0$ at $2t - 6 = 0$, i.e. $t = 3$ **B1**.

Exam-style 3.1 ▪ 1 mark

The total distance travelled over $[a, b]$ is

A $\int_a^b v\,dt$

B $\int_a^b |v|\,dt$

C $v(b) - v(a)$

D $\int_a^b a\,dt$

Solution 3.1

Method: Total distance

A $\int_a^b v\,dt$

B $\int_a^b |v|\,dt$ ✓

C $v(b) - v(a)$

D $\int_a^b a\,dt$

Worked steps

B — total distance is $\int |v|\,dt$.

Exam-style 3.2 ▪ 2 marks

A particle moves with velocity $v(t) = 3t^2 - 12$ m/s.
(a) Find the displacement over $0 \leq t \leq 3$.
(b) At what time does the particle change direction?

Solution 3.2

Method: Displacement

Worked steps

(a) $\int_0^3 (3t^2 - 12)\, dt = \left[t^3 - 12t\right]_0^3 = 27 - 36 = -9$ m **M1** **A1**. (b) $v = 0$: $3t^2 = 12 \Rightarrow t = 2$ **M1** **A1**.

Exam-style 3.3 ▪ 2 marks

A particle has velocity $v(t) = t - 3$ on $0 \leq t \leq 5$.
(a) Find the displacement.
(b) Find the total distance travelled.

Solution 3.3

Method: Total distance

Worked steps

(a) $\int_0^5 (t-3)\,dt = \left[\frac{t^2}{2} - 3t\right]_0^5 = 12.5 - 15 = -2.5$ **M1** **A1**. (b) $v < 0$ on $[0, 3)$, $v > 0$ on $(3, 5]$; distance $= \left| \int_0^3 \right| + \int_3^5 = |-4.5| + 2 = 6.5$ **M1** **M1** **A1** *split*.