

Exercise walk-through

Finding the Average Value of a Function on an Interval ■ 8.1

eXercise

You must be able to

- apply $f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$
- distinguish the **average value** 平均值 of a function from the average **rate of change**
- interpret the result in context

Method — The average-value formula

The average value of f over $[a, b]$ is

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) \, dx.$$

It is the constant height of a rectangle with the same area as the region under f .

Method — A worked value

Average of $f(x) = x^2$ on $[0, 3]$: $\frac{1}{3-0} \int_0^3 x^2 dx = \frac{1}{3} \left[\frac{x^3}{3} \right]_0^3 = \frac{1}{3} \cdot 9 = 3.$

Method — Average value vs average rate of change

- **Average value** of f uses an **integral**: $\frac{1}{b-a} \int_a^b f$.
- **Average rate of change** of f uses a **difference**: $\frac{f(b) - f(a)}{b - a}$.

Read the question carefully to pick the right one.

Method — In context

If $v(t)$ is speed, its average value $\frac{1}{b-a} \int_a^b v \, dt$ is the **average speed** over the interval — total distance divided by time.

Practice 2.1 ■ 1 mark

Write the formula for the average value of f on $[2, 6]$.

Solution 2.1

Method: The average-value formula

Worked steps

$$\frac{1}{6-2} \int_2^6 f(x) \, dx \quad \text{B1}.$$

Practice 2.2 ■ 3 marks

Find the average value of $f(x) = 4x$ on $[0, 5]$.

Solution 2.2

Worked steps

$$\frac{1}{5} \int_0^5 4x \, dx = \frac{1}{5} [2x^2]_0^5 \quad \text{M1} = \frac{1}{5} (50) \quad \text{M1} = 10 \quad \text{A1}.$$

Practice 2.3 ■ 3 marks

Find the average value of $f(x) = 6x^2$ on $[0, 2]$.

Solution 2.3

Worked steps

$$\frac{1}{2} \int_0^2 6x^2 dx = \frac{1}{2} [2x^3]_0^2 \quad \text{M1} = \frac{1}{2}(16) \quad \text{M1} = 8 \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

The average value of f on $[a, b]$ is

A $f(b) - f(a)$

B $\frac{f(b) - f(a)}{b - a}$

C $\frac{1}{b - a} \int_a^b f dx$

D $\int_a^b f dx$

Solution 3.1

A $f(b) - f(a)$

B $\frac{f(b) - f(a)}{b - a}$

C $\frac{1}{b - a} \int_a^b f dx$



D $\int_a^b f dx$

Worked steps

C — the average value is the integral divided by the width.

Exam-style 3.2 ■ 3 marks

A tank's inflow rate is $r(t) = 12 - t$ litres/min for $0 \leq t \leq 8$.

- (a) Find the average inflow rate over this interval.
- (b) State the units of your answer.

Solution 3.2

Method: In context

Worked steps

(a) $\frac{1}{8} \int_0^8 (12 - t) dt = \frac{1}{8} \left[12t - \frac{t^2}{2} \right]_0^8 = \frac{1}{8} (96 - 32) = 8$ **M1 M1 A1**. (b) litres per minute **B1**.

Exam-style 3.3 ■ 4 marks

The temperature over a day is modelled by $T(t) = 20 + 6t - t^2$ for $0 \leq t \leq 6$ hours.
Find the average temperature over the interval.

Solution 3.3

Method: In context

Worked steps

$$\frac{1}{6} \int_0^6 (20 + 6t - t^2) dt = \frac{1}{6} [20t + 3t^2 - \frac{t^3}{3}]_0^6 \quad \text{M1} \quad \text{M1} = \frac{1}{6} (120 + 108 - 72) = \frac{1}{6} (156)$$

$\text{M1} = 26^\circ \quad \text{A1}.$