Exercise walk-through

Finding Particular Solutions Using Initial Conditions and Separation of Variables ▪ 7.7

eXercise

You must be able to

- find the **general solution** by separation of variables
- substitute the **initial condition 初始条件** to solve for C
- write the **particular solution** and use it to predict a value

Method — General, then particular

Solve $\frac{dy}{dx} = 2x$ with $y(0) = 3$. Integrate: $y = x^2 + C$ (general). Apply the condition $y(0) = 3$: $3 = 0 + C$, so $C = 3$. Particular solution: $y = x^2 + 3$.

Method — Finding C after an exponential solution

$\dfrac{dy}{dt} = ky$ has general solution $y = Ae^{kt}$. If $y(0) = 5$, then $5 = Ae^0 = A$, so $A = 5$ and $y = 5e^{kt}$. The initial value **is** the constant A here.

Method — Substituting a non-zero start

Solve $\dfrac{dy}{dx} = \dfrac{x}{y}$, $y(0) = 2$. Separating gives $y^2 = x^2 + C_1$. At $(0, 2)$: $4 = 0 + C_1$, so $y^2 = x^2 + 4$, i.e. $y = \sqrt{x^2 + 4}$ (positive root, to match $y(0) = 2 > 0$).

Method — Using the particular solution

Once you have $y = \sqrt{x^2 + 4}$, predict any value: $y(3) = \sqrt{9 + 4} = \sqrt{13}$.

Practice 2.1 ▪ 3 marks

Solve $\dfrac{dy}{dx} = 4x$ with $y(1) = 5$.

Solution 2.1

Worked steps

$y = 2x^2 + C$ **M1**; $5 = 2 + C \Rightarrow C = 3$ **M1**; $y = 2x^2 + 3$ **A1**.

Practice 2.2 ▪ 1 mark

For $y = Ae^{2t}$ with $y(0) = 7$, find A.

Solution 2.2

Worked steps

$A = 7$ $\boxed{\textbf{B1}}$.

Practice 2.3 ▪ 3 marks

Solve $\dfrac{dy}{dx} = 3x^2$ with $y(0) = -1$.

Solution 2.3

Worked steps

$y = x^3 + C$ **M1**; $-1 = 0 + C \Rightarrow C = -1$ **M1**; $y = x^3 - 1$ **A1**.

Exam-style 3.1 ▪ 1 mark

The general solution $y = x^2 + C$ passes through $(2, 10)$. Then $C =$

- **A** 2
- **B** 6
- **C** 10
- **D** 14

Solution 3.1

A 2

B 6 ✓

C 10

D 14

Worked steps

B — $10 = 4 + C \Rightarrow C = 6.$

Exam-style 3.2 ▪ 2 marks

A population satisfies $\dfrac{dP}{dt} = 0.5P$ with $P(0) = 200$.

(a) Write the general solution.

(b) Find the particular solution using the initial condition.

(c) Predict P at $t = 4$.

Solution 3.2

Method: General, then particular

Worked steps

(a) $P = Ae^{0.5t}$ **M1** **A1**. (b) $P(0) = A = 200$, so $P = 200e^{0.5t}$ **B1**. (c) $P(4) = 200e^{2} \approx 1478$ **M1** **A1**.

Exam-style 3.3 ▪ 4 marks

Solve $\dfrac{dy}{dx} = \dfrac{6x}{y}$ with $y(0) = 4$, giving y explicitly.

Solution 3.3

Worked steps

$y\,dy = 6x\,dx$ **M1**; $\frac{1}{2}y^2 = 3x^2 + C$ **M1**; at $(0,4)$: $8 = C$ **M1**; $y = \sqrt{6x^2+16}$ **A1**.