Exercise walk-through

Finding General Solutions Using Separation of Variables ▪ 7.6

eXercise

You must be able to

- separate a **separable 可分离** equation so each variable is on its own side
- integrate both sides and include a single constant C
- solve for y where the question asks for an explicit solution

Method — Separating the variables

For $\dfrac{dy}{dx} = xy$: put all y with dy and all x with dx:

$$\frac{1}{y}\,dy = x\,dx.$$

Method — Integrating both sides

$$\int \frac{1}{y}\, dy = \int x\, dx \;\Rightarrow\; \ln|y| = \frac{x^2}{2} + C.$$

One constant C is enough (combine the two).

Method — Solving for y

Exponentiate: $|y| = e^{x^2/2+C} = e^C e^{x^2/2}$. Writing $A = \pm e^C$ (a new constant),

$$y = Ae^{x^2/2}.$$

Method — A quick separable example

$$\frac{dy}{dx} = \frac{x}{y}\text{: } y\,dy = x\,dx\text{, so } \tfrac{1}{2}y^2 = \tfrac{1}{2}x^2 + C\text{, i.e. } y^2 = x^2 + C_1.$$

Practice 2.1 ▪ 1 mark

Separate the variables in $\dfrac{dy}{dx} = 2xy$.

Solution 2.1

Method: Separating the variables

Worked steps

$$\frac{1}{y}\,dy = 2x\,dx \quad \boxed{\textbf{B1}}.$$

Practice 2.2 ▪ 3 marks

Solve $\dfrac{dy}{dx} = \dfrac{1}{y}$ (find the general solution).

Solution 2.2

Worked steps

$y\,dy = dx$ **M1**; $\frac{1}{2}y^2 = x + C$ **M1**; $y^2 = 2x + C_1$ **A1**.

Practice 2.3 ▪ 3 marks

Solve $\dfrac{dy}{dx} = 3y$ (general solution in terms of a constant A).

Solution 2.3

Worked steps

$\frac{1}{y}\,dy = 3\,dx$ **M1**; $\ln|y| = 3x + C$ **M1**; $y = Ae^{3x}$ **A1**.

Exam-style 3.1 ▪ 1 mark

After separating $\dfrac{dy}{dx} = ky$, integrating the left side gives

A y

B $\ln|y|$

C $\dfrac{1}{y^2}$

D e^y

Solution 3.1

Method: Separating the variables

A y

B $\ln|y|$ ✓

C $\dfrac{1}{y^2}$

D e^y

Worked steps

B — $\int \frac{1}{y}\,dy = \ln|y|$.

Exam-style 3.2 ▪ 3 marks

Solve $\dfrac{dy}{dx} = xy^2$.

(a) Separate and integrate both sides.

(b) Solve for y in terms of x and a constant.

Solution 3.2

Method: Integrating both sides

Worked steps

(a) $\frac{1}{y^2}\,dy = x\,dx$ **M1**; $-\frac{1}{y} = \frac{x^2}{2} + C$ **M1** **A1**. (b) $y = -\frac{1}{\frac{x^2}{2} + C} = \frac{-2}{x^2 + C_1}$ **M1** **A1**.

Exam-style 3.3 ▪ 3 marks

Solve $\dfrac{dy}{dx} = \dfrac{\cos x}{y}$, giving the general solution.

Solution 3.3

Worked steps

$y\,dy = \cos x\,dx$ **M1**; $\frac{1}{2}y^2 = \sin x + C$ **M1**; $y^2 = 2\sin x + C_1$ **A1**.