Exercise walk-through

Sketching Slope Fields ▪ 7.3

eXercise

You must be able to

- compute the slope $\frac{dy}{dx}$ at grid points and draw short segments (a **slope field** 斜率场)
- match a differential equation to its slope field
- sketch a solution curve that **follows** the segments through a given point

Method — Computing a slope at a point

For $\frac{dy}{dx} = x + y$, the slope at $(1, 2)$ is $1 + 2 = 3$: draw a short segment of slope 3 there. Repeating at many points gives the slope field.

Method — Slopes that depend only on x

If $\frac{dy}{dx} = 2x$, the slope depends on x **only**, so every segment in a vertical column has the same slope — the field looks the same as you move up or down.

Method — Slopes that depend only on y

If $\frac{dy}{dx} = y$, segments in a horizontal row match. Where $y = 0$ the slope is 0 (horizontal segments along the x-axis).

Method — Following the field

A solution curve is tangent to the segments everywhere. Starting at a point, draw a curve that always runs **along** the nearby segments — like a boat following a current.

Practice 2.1 ▪ 1 mark

For $\frac{dy}{dx} = x - y$, find the slope at $(3, 1)$.

Solution 2.1

Worked steps

$3 - 1 = 2$ B1.

Practice 2.2 ▪ 1 mark

For $\frac{dy}{dx} = xy$, find the slope at $(2, -1)$.

Solution 2.2

Worked steps

$(2)(-1) = -2$ B1.

Practice 2.3 ▪ 1 mark

For $\frac{dy}{dx} = y - 1$, state where all the segments are **horizontal**.

Solution 2.3

Method: Slopes that depend only on y

Worked steps

Where $y - 1 = 0$, i.e. along the line $y = 1$ B1.

Exam-style 3.1 ▪ 1 mark

In the slope field for $\dfrac{dy}{dx} = 2x$, along the line $x = 0$ the segments are

A vertical

B horizontal

C slope $+1$

D slope $+2$

Solution 3.1

Method: Slopes that depend only on y

A vertical

B horizontal ✓

C slope $+1$

D slope $+2$

Worked steps

B — at $x = 0$ the slope is $2(0) = 0$, horizontal.

Exam-style 3.2 ▪ 3 marks

A slope field is given by $\dfrac{dy}{dx} = x + 1$.

(a) Find the slope at the points $(-1, 0)$, $(0, 2)$, and $(2, -3)$.

(b) Along which vertical line are all segments horizontal?

Solution 3.2

Method: Slopes that depend only on y

Worked steps

(a) $(-1, 0)$: 0; $(0, 2)$: 1; $(2, -3)$: 3 B1 *each.* (b) $x = -1$ (where $x + 1 = 0$) B1.

Exam-style 3.3 ▪ 2 marks

A slope field has horizontal segments only along the x-axis ($y = 0$) and gets steeper as $|y|$ grows, with the same pattern in every vertical column. Which differential equation fits: $\frac{dy}{dx} = x$, $\frac{dy}{dx} = y$, or $\frac{dy}{dx} = xy$? Explain.

Solution 3.3

Method: Slopes that depend only on x

Worked steps

$\dfrac{dy}{dx} = y$ — the slope is 0 where $y = 0$ and grows with $|y|$, and depends on y only, so it is the same in every column **M1** **A1** *reasoning, answer.*