Exercise walk-through

Verifying Solutions for Differential Equations ▪ 7.2

eXercise

You must be able to

- differentiate a proposed solution and **substitute** it into the equation
- confirm both sides are equal for all x (or t)
- check that an **initial condition** **初始条件** is also met

Method — Substitute and compare

To verify $y = e^{2x}$ solves $\frac{dy}{dx} = 2y$: differentiate, $\frac{dy}{dx} = 2e^{2x}$; the right side is $2y = 2e^{2x}$. They match, so $y = e^{2x}$ **is** a solution.

Method — A solution with a constant

$y = Ce^{kt}$ solves $\frac{dy}{dt} = ky$ for **any** constant C: $\frac{dy}{dt} = kCe^{kt} = ky$. This is the general solution; each C gives one curve.

Method — Checking an implicit solution

To check $y^2 = x^2 + C$ solves $\frac{dy}{dx} = \frac{x}{y}$: differentiate implicitly, $2y\frac{dy}{dx} = 2x$, so $\frac{dy}{dx} = \frac{x}{y}$. Verified.

Method — Including an initial condition

If asked to verify a **particular** solution, also check the point. $y = 3e^{2x}$ with $y(0) = 3$: at $x = 0$, $y = 3e^0 = 3$. The condition holds.

Practice 2.1 ▪ 2 marks

Verify that $y = e^{5x}$ satisfies $\dfrac{dy}{dx} = 5y$.

Solution 2.1

Method: Substitute and compare

Worked steps

$\frac{dy}{dx} = 5e^{5x}$ M1; this equals $5y = 5e^{5x}$, so it is a solution A1.

Practice 2.2 ▪ 2 marks

Show that $y = 4e^{-t}$ satisfies $\dfrac{dy}{dt} = -y$.

Solution 2.2

Worked steps

$\frac{dy}{dt} = -4e^{-t}$ M1; and $-y = -4e^{-t}$, so they match A1.

Practice 2.3 ▪ 1 mark

For $y = Ce^{3x}$, state $\frac{dy}{dx}$ in terms of y.

Solution 2.3

Worked steps

$\frac{dy}{dx} = 3Ce^{3x} = 3y$ B1.

Exam-style 3.1 ▪ 1 mark

Which function satisfies $\frac{dy}{dx} = 2y$?

A $y = 2x$

B $y = x^2$

C $y = e^{2x}$

D $y = 2e^x$

Solution 3.1

A $y = 2x$

B $y = x^2$

C $y = e^{2x}$ ✓

D $y = 2e^x$

Worked steps

C — $\dfrac{d}{dx} e^{2x} = 2e^{2x} = 2y.$

Exam-style 3.2 ▪ 1 mark

Consider $y = x^2 + 3x + C$.

(a) Show that $\dfrac{dy}{dx} = 2x + 3$.

(b) Find the value of C so that the solution passes through $(1, 6)$.

Solution 3.2

Worked steps

(a) $\dfrac{dy}{dx} = 2x + 3$ B1. (b) $6 = 1 + 3 + C \Rightarrow C = 2$ M1 A1.

Exam-style 3.3 ▪ 3 marks

Verify that $y = \sin(2x)$ satisfies $\dfrac{d^2y}{dx^2} = -4y$.

Solution 3.3

Method: Substitute and compare

Worked steps

$\frac{dy}{dx} = 2\cos(2x)$ M1; $\frac{d^2y}{dx^2} = -4\sin(2x)$ M1; this equals $-4y = -4\sin(2x)$ A1.