Exercise walk-through

Integrating Using Substitution ▪ 6.9

eXercise

You must be able to

- choose $u = g(x)$ so that its derivative $g'(x)$ also appears in the integrand
- convert dx to du and integrate in terms of u
- handle a **definite** integral by changing the limits or converting back to x

Method — Choosing u

Look for an "inside function" whose derivative is also present. For $\int 2x\,(x^2+1)^3\,dx$, let $u = x^2+1$, so $du = 2x\,dx$. The integral becomes

$$\int u^3\,du = \frac{u^4}{4} + C = \frac{(x^2+1)^4}{4} + C.$$

Method — Adjusting for a constant

If the derivative is off by a constant, fix it. For $\int x\,(x^2+1)^3\,dx$, let $u = x^2+1$, $du = 2x\,dx$, so $x\,dx = \frac{1}{2}\,du$:

$$\int u^3 \cdot \tfrac{1}{2}\,du = \frac{u^4}{8} + C.$$

Method — Definite integral — change the limits

For $\int_0^2 2x(x^2+1)^3\,dx$, with $u = x^2+1$: when $x=0$, $u=1$; when $x=2$, $u=5$. So

$$\int_1^5 u^3\,du = \left[\tfrac{u^4}{4}\right]_1^5 = \frac{625-1}{4} = 156.$$

Method — A trig example

$\int \cos(3x)\,dx$: let $u = 3x$, $du = 3\,dx$, so $dx = \frac{1}{3}\,du$, giving $\frac{1}{3}\sin(3x) + C$.

Practice 2.1 ▪ 2 marks

For $\int 3x^2(x^3+5)^4\,dx$, state a good choice of u and find du.

Solution 2.1

Worked steps

$u = x^3 + 5$ **M1**; $du = 3x^2\,dx$ **A1**.

Practice 2.2 ▪ 2 marks

Find $\displaystyle\int 2x\, e^{x^2}\, dx$.

Solution 2.2

Worked steps

$u = x^2,\ du = 2x\,dx$ **M1**; $\int e^u\,du = e^{x^2} + C$ **A1**.

Practice 2.3 ▪ 2 marks

Find $\displaystyle\int \sin(4x)\, dx$.

Solution 2.3

Worked steps

$u = 4x,\ dx = \frac{1}{4}du;\ -\frac{1}{4}\cos(4x) + C$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

To integrate $\int x\cos(x^2)\,dx$, the best substitution is

A $u = x$

B $u = \cos x$

C $u = x^2$

D $u = \cos(x^2)$

Solution 3.1

A $u = x$

B $u = \cos x$

C $u = x^2$ ✓

D $u = \cos(x^2)$

Worked steps

C — $u = x^2$ gives $du = 2x\,dx$, matching the x in the integrand.

Exam-style 3.2 ▪ 3 marks

Find $\displaystyle\int (2x+1)(x^2+x)^5\,dx.$

Solution 3.2

Worked steps

$u = x^2 + x,\ du = (2x+1)\,dx$ **M1**; $\int u^5\,du = \frac{u^6}{6}$ **M1** $= \dfrac{(x^2+x)^6}{6} + C$ **A1**.

Exam-style 3.3 ▪ 4 marks

Evaluate $\int_0^1 3x^2\sqrt{x^3+1}\,dx$ using a substitution, changing the limits.

Solution 3.3

Method: Definite integral — change the limits

Worked steps

$u = x^3 + 1,\ du = 3x^2\,dx$ M1; limits $x = 0 \to u = 1,\ x = 1 \to u = 2$ M1;
$\int_1^2 \sqrt{u}\,du = \left[\frac{2}{3}u^{3/2}\right]_1^2$ M1 $= \frac{2}{3}(2\sqrt{2} - 1) \approx 1.22$ A1.