Exercise walk-through

Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation ▪ 6.8

eXercise

You must be able to

- apply the **power rule for integration** $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
- integrate $\frac{1}{x}$, e^x, and basic trig functions
- always include the **constant of integration 积分常数** C

Method — The power rule for integration

Add one to the power and divide by the new power:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1).$$

So $\int x^3 \, dx = \frac{x^4}{4} + C$ and $\int 5 \, dx = 5x + C$.

Method — The special case n = -1

The power rule fails for $n = -1$; instead $\displaystyle\int \frac{1}{x}\, dx = \ln|x| + C$.

Method — Standard antiderivatives

$$\int e^x\,dx = e^x + C,\ \int \cos x\,dx = \sin x + C,\ \int \sin x\,dx = -\cos x + C,$$
$$\int \sec^2 x\,dx = \tan x + C.$$

Method — Term by term (and never forget +C)

Integrate a sum piece by piece: $\int (6x^2 - 4x + 1)\, dx = 2x^3 - 2x^2 + x + C$. The $+C$ is essential — an indefinite integral is a **family** of functions.

Practice 2.1 ▪ 1 mark

Find $\displaystyle\int x^5 \, dx$.

Solution 2.1

Worked steps

$$\frac{x^6}{6} + C \quad \boxed{\textbf{B1}}.$$

Practice 2.2 ▪ 2 marks

Find $\int (4x^3 - 2x)\,dx$.

Solution 2.2

Worked steps

$x^4 - x^2 + C$ **M1** **A1**.

Practice 2.3 ▪ 2 marks

Find $\int \left(\frac{1}{x} + e^x\right)\, dx$.

Solution 2.3

Worked steps

$\ln|x| + e^x + C$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

$$\int \cos x \, dx =$$

A $-\sin x + C$

B $\sin x + C$

C $-\cos x + C$

D $\tan x + C$

Solution 3.1

A $-\sin x + C$

B $\sin x + C$ ✓

C $-\cos x + C$

D $\tan x + C$

Worked steps

B — $\int \cos x \, dx = \sin x + C$.

Exam-style 3.2 ▪ 2 marks

(a) Find the general antiderivative of $f(x) = 3x^2 + 4x$.
(b) Find the particular antiderivative F with $F(0) = 5$.

Solution 3.2

Worked steps

(a) $F(x) = x^3 + 2x^2 + C$ **M1** **A1**. (b) $F(0) = C = 5$, so $F(x) = x^3 + 2x^2 + 5$ **M1** **A1**.

Exam-style 3.3 ▪ 3 marks

Find $\int \left(\sqrt{x} + \frac{2}{x} \right) \, dx$. Write $\sqrt{x}$ as $x^{1/2}$ first.

Solution 3.3

Worked steps

$\int (x^{1/2} + 2x^{-1})\,dx = \dfrac{x^{3/2}}{3/2} + 2\ln|x| + C = \frac{2}{3}x^{3/2} + 2\ln|x| + C$ **M1** **M1** **A1** *rewrite, integrate.*