Exercise walk-through

The Fundamental Theorem of Calculus and Definite Integrals ▪ 6.7

eXercise

You must be able to

- apply $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$
- evaluate integrals of powers, exponentials, and basic trig functions
- interpret $\int_a^b f'(x)\,dx = f(b) - f(a)$ as a **net change 净变化**

Method — Evaluating with an antiderivative

$\int_1^3 2x\,dx$: an antiderivative of $2x$ is x^2, so

$$\int_1^3 2x\,dx = \left[x^2\right]_1^3 = 3^2 - 1^2 = 8.$$

Method — A power and a constant

$\int_0^2 (3x^2 + 1)\,dx = \left[x^3 + x\right]_0^2 = (8 + 2) - 0 = 10$. Antidifferentiate term by term, then subtract the values at the limits.

Method — Net change from a rate

If f' is a rate of change, then $\int_a^b f'(x)\,dx = f(b) - f(a)$ is the **net change** in f. If a tank's level changes at rate $r(t)$ and $\int_0^{10} r\,dt = -4$, the level **fell** by 4 over those 10 units.

Method — Trig and exponential

$$\int_0^{\pi} \sin x \, dx = \big[-\cos x \big]_0^{\pi} = -\cos \pi + \cos 0 = 1 + 1 = 2, \text{ and } \int_0^{1} e^x \, dx = \big[e^x \big]_0^{1} = e - 1.$$

Practice 2.1 ▪ 2 marks

Evaluate $\int_0^4 x\,dx$.

Solution 2.1

Worked steps

$\left[\frac{1}{2}x^2\right]_0^4 = 8 - 0 = 8$ M1 A1.

Practice 2.2 ▪ 2 marks

Evaluate $\int_1^2 (4x^3)\,dx$.

Solution 2.2

Worked steps

$\left[x^4\right]_1^2 = 16 - 1 = 15$ M1 A1.

Practice 2.3 ▪ 2 marks

Evaluate $\int_0^{\pi/2} \cos x \, dx$.

Solution 2.3

Worked steps

$\left[\sin x\right]_0^{\pi/2} = 1 - 0 = 1$ M1 A1.

Exam-style 3.1 ▪ 1 mark

$$\int_a^b f'(x)\,dx =$$

A $f'(b) - f'(a)$

B $f(b) - f(a)$

C $f(b) + f(a)$

D $\frac{1}{2}(f(a) + f(b))$

Solution 3.1

A $f'(b) - f'(a)$

B $f(b) - f(a)$ ✓

C $f(b) + f(a)$

D $\frac{1}{2}(f(a) + f(b))$

Worked steps

B — by the FTC, $\int_a^b f' = f(b) - f(a)$.

Exam-style 3.2 ▪ 2 marks

A particle's velocity is $v(t) = 3t^2 - 4$ m/s.

(a) Evaluate $\displaystyle\int_0^2 v(t)\,dt$.

(b) State what this value represents physically.

Solution 3.2

Worked steps

(a) $\left[t^3 - 4t\right]_0^2 = (8 - 8) - 0 = 0$ M1 A1. (b) The particle's **displacement** over $0 \le t \le 2$ is 0 — it returns to its start B1.

Exam-style 3.3 ▪ 3 marks

Water enters a tank at rate $r(t) = 6t$ L/min. Find the total volume added between $t = 1$ and $t = 4$ minutes.

Solution 3.3

Method: Net change from a rate

Worked steps

$$\int_1^4 6t\,dt = \left[3t^2\right]_1^4 \ \text{M1} = 48 - 3 \ \text{M1} = 45 \text{ L} \ \text{A1}.$$