

Exercise walk-through

Applying Properties of Definite Integrals ■ 6.6

eXercise

You must be able to

- use $\int_a^a f = 0$ and $\int_b^a f = -\int_a^b f$
- **split** an integral over adjacent intervals: $\int_a^c f = \int_a^b f + \int_b^c f$
- factor out constants and add integrands: $\int_a^b (cf + g) = c \int_a^b f + \int_a^b g$

Method — Zero-width and reversing

$$\int_a^a f dx = 0 \text{ (no width). Reversing the limits flips the sign: } \int_b^a f dx = - \int_a^b f dx.$$

Method — Adding adjacent intervals

Integrals join end-to-end: $\int_1^4 f = \int_1^3 f + \int_3^4 f$. If $\int_1^3 f = 5$ and $\int_1^4 f = 9$, then $\int_3^4 f = 9 - 5 = 4$.

Method — Linearity

Constants come out and sums split:

$$\int_a^b (3f(x) + 2g(x)) \, dx = 3 \int_a^b f + 2 \int_a^b g.$$

If $\int_0^2 f = 4$ and $\int_0^2 g = 1$, then $\int_0^2 (3f + 2g) = 3(4) + 2(1) = 14$.

Method — Combining with a reversal

Given $\int_2^5 f = 10$, find $\int_5^2 3f$. Reverse and factor: $\int_5^2 3f = 3 \int_5^2 f = 3(-10) = -30$.

Practice 2.1 ■ 1 mark

Find $\int_0^7 f dx$.

Solution 2.1

Worked steps

$$\int_0^7 f = \int_0^3 f + \int_3^7 f = 6 + (-2) = 4 \quad \text{B1}.$$

Practice 2.2 ■ 1 mark

Find $\int_3^0 f dx$.

Solution 2.2

Worked steps

$$\int_3^0 f = -\int_0^3 f = -6 \quad \text{B1}.$$

Practice 2.3 ■ 2 marks

Find $\int_0^3 (2f + g) \, dx$.

Solution 2.3

Worked steps

$$2(6) + 5 = 17 \quad \text{M1} \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

$$\int_5^5 f(x) \, dx =$$

A $f(5)$

B 1

C 0

D undefined

Solution 3.1

A $f(5)$

B 1

C 0



D undefined

Worked steps

C — a zero-width interval gives 0.

Exam-style 3.2 ■ 2 marks

You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(a) Find $\int_4^6 h \, dx$.

(b) Find $\int_6^1 h \, dx$.

Solution 3.2

Worked steps

$$(a) \int_4^6 h = \int_1^6 h - \int_1^4 h = 12 - 7 = 5 \quad \text{M1 A1.} \quad (b) \int_6^1 h = -12 \quad \text{B1.}$$

Exam-style 3.3 ■ 2 marks

Given $\int_0^2 p = 8$ and $\int_0^2 q = -3$, find $\int_0^2 (5p - 4q) dx$.

Solution 3.3

Worked steps

$$5(8) - 4(-3) = 40 + 12 = 52 \quad \text{M1 A1}.$$