Exercise walk-through

Interpreting the Behavior of Accumulation Functions Involving Area ▪ 6.5

eXercise

You must be able to

- decide where g is **increasing/decreasing** from the sign of f
- locate **local extrema 极值** of g where f crosses zero (sign change)
- decide **concavity** of g from whether f is increasing or decreasing

Method — g rises where f is positive

Because $g'(x) = f(x)$, the accumulation function g **increases** on intervals where the graph of f is **above** the axis and **decreases** where f is **below** the axis.

Method — Extrema of g at zeros of f

g has a **local maximum** where f changes from $+$ to $-$, and a **local minimum** where f changes from $-$ to $+$. A zero of f where the sign does **not** change is not an extremum of g.

Method — Concavity of g from the slope of f

$g''(x) = f'(x)$, so g is **concave up** where f is **increasing** and **concave down** where f is **decreasing**. A point where f has a local max or min is a **point of inflection** of g.

Method — Comparing values of g

To compare $g(3)$ and $g(0)$, look at the net signed area of f between them: $g(3) - g(0) = \int_0^3 f\,dt$. More area above the axis than below makes $g(3) > g(0)$.

Practice 2.1 ▪ 1 mark

On an interval where $f(t) < 0$, state whether g is increasing or decreasing.

Solution 2.1

Method: Concavity of g from the slope of f

Worked steps

Decreasing — $g'(x) = f(x) < 0$ **B1**.

Practice 2.2 ▪ 1 mark

At a point where f changes from positive to negative, what feature does g have?

Solution 2.2

Method: Extrema of g at zeros of f

Worked steps

A local maximum **B1**.

Practice 2.3 ▪ 1 mark

On an interval where f is increasing, state the concavity of g.

Solution 2.3

Method: Concavity of g from the slope of f

Worked steps

Concave up — $g''(x) = f'(x) > 0$ **B1**.

Exam-style 3.1 ▪ 1 mark

$g(x) = \int_0^x f(t)\,dt$. The graph of f crosses from below the axis to above it at $x = 4$. At $x = 4$, g has a

- **A** local maximum
- **B** local minimum
- **C** point of inflection
- **D** vertical asymptote

Solution 3.1

Method: g rises where f is positive

A local maximum

B local minimum ✓

C point of inflection

D vertical asymptote

Worked steps

B — f changes $-$ to $+$, so g' changes sign the same way and g has a local minimum.

Exam-style 3.2 ▪ 1 mark

The graph of f consists of straight segments: $f > 0$ on $(0, 3)$, $f < 0$ on $(3, 7)$, and $f(3) = 0$. Let $g(x) = \int_0^x f(t)\,dt$.

(a) On what interval is g increasing?

(b) At what x does g attain a local maximum? Justify.

Solution 3.2

Method: Extrema of g at zeros of f

Worked steps

(a) g increases on $(0, 3)$ where $f > 0$ **B1**. (b) At $x = 3$ **M1**, because f (which is g') changes from positive to negative there **A1**.

Exam-style 3.3 ▪ 2 marks

For $g(x) = \int_0^x f(t)\,dt$, the graph of f is increasing on $(1,5)$. State, with a reason, the concavity of g on $(1,5)$.

Solution 3.3

Method: Concavity of g from the slope of f

Worked steps

Concave up — $g''(x) = f'(x) > 0$ since f is increasing **M1** **A1**.