Exercise walk-through

Selecting Techniques for Antidifferentiation ▪ 6.14

eXercise

You must be able to

- recognise when a **basic rule**, a **substitution**, **long division**, or **completing the square** applies
- match an integrand's shape to the fastest method
- carry the chosen method through to a correct antiderivative

Method — The decision guide

- **Basic rule** — a power, e^x, $\sin$ / $\cos$, or $\frac{1}{x}$ on its own.
- **Substitution** — an inside function whose derivative also appears (e.g. $2x$ beside x^2+1).
- **Long division** — a rational function with numerator degree $\geq$ denominator degree.
- **Completing the square** — an irreducible quadratic $x^2 + bx + c$ in a denominator (arctangent).

Method — Spotting a substitution

$\int x^2 e^{x^3}\,dx$: the x^2 is (up to a constant) the derivative of x^3 — **substitute** $u = x^3$.

Answer $\frac{1}{3}e^{x^3} + C$.

Method — Spotting a division

$\int \frac{x^3}{x^2+1}\,dx$: top-heavy, so **divide** first: $x - \frac{x}{x^2+1}$, then integrate (the second piece needs $u = x^2 + 1$).

Method — Spotting a basic rule

$\int (2x + \cos x)\, dx$ needs no tricks — just the **basic rules**: $x^2 + \sin x + C$.

Practice 2.1 ▪ 1 mark

$$\int \frac{1}{x^2 + 4}\, dx$$

Solution 2.1

Method: Spotting a division

Worked steps

Complete-the-square / arctangent form (basic arctangent rule) B1.

Practice 2.2 ▪ 1 mark

$$\int 5x^4(x^5+2)^7\,dx$$

Solution 2.2

Worked steps

Substitution, $u = x^5 + 2$ B1.

Practice 2.3 ▪ 1 mark

$$\int \frac{x^2+1}{x}\,dx$$

Solution 2.3

Method: Spotting a division

Worked steps

Basic — split into $x + \frac{1}{x}$ first B1.

Exam-style 3.1 ▪ 1 mark

Which integral is best done by **substitution**?

A $\displaystyle\int (x^3 - 2)\,dx$

B $\displaystyle\int \frac{1}{x^2 + 9}\,dx$

C $\displaystyle\int \cos(x^2) \cdot 2x\,dx$

D $\displaystyle\int \frac{x^2}{x + 1}\,dx$

Solution 3.1

A $\int (x^3 - 2)\,dx$

B $\int \dfrac{1}{x^2 + 9}\,dx$

C $\int \cos(x^2) \cdot 2x\,dx$ ✓

D $\int \dfrac{x^2}{x+1}\,dx$

Worked steps

C — $2x$ is the derivative of x^2, so substitute $u = x^2$.

Exam-style 3.2 ▪ 3 marks

Find $\int \frac{x^2+3}{x}\,dx$ (rewrite the fraction first).

Solution 3.2

Method: Spotting a division

Worked steps

$\frac{x^2+3}{x} = x + \frac{3}{x}$ M1; $\int \left(x + \frac{3}{x}\right) dx = \frac{x^2}{2} + 3\ln|x| + C$ M1 A1.

Exam-style 3.3 ▪ 3 marks

Find $\int \frac{6x}{x^2+1}\,dx$.

Solution 3.3

Method: Spotting a division

Worked steps

$u = x^2+1$, $du = 2x\,dx$, so $6x\,dx = 3\,du$ M1; $\int \frac{3}{u}\,du = 3\ln|u|$ M1 $= 3\ln(x^2+1)+C$ A1.