Exercise walk-through

Exploring Accumulations of Change ▪ 6.1

eXercise

You must be able to

- interpret the **area** under a rate-of-change graph as an **accumulated change** 累积变化
- treat area **above** the axis as positive and area **below** as negative
- attach correct **units** (rate unit $\times$ time unit) to an accumulation

Method — Area under a rate graph is a total

If water flows in at a **rate** $R(t)$ litres per minute, the total water added between $t = 0$ and $t = 5$ is the **area** under the R-vs-t graph over $[0, 5]$. A rate graph that is a rectangle of height 3 L/min over 5 min accumulates $3 \times 5 = 15$ L.

Method — Splitting into simple shapes

If the rate graph is made of straight lines, find the area with triangles and rectangles. A rate rising linearly from 0 to 4 L/min over 6 min is a triangle: area $= \frac{1}{2}(6)(4) = 12$ L.

Method — Signed area

When a rate goes **negative** (water leaving), that area counts as a **loss**. Over a stretch where the graph is $+8$ area then -3 area, the net accumulated change is $8 - 3 = +5$.

Method — Units

The accumulation's unit is (rate unit) $\times$ (time unit). A rate in L/min over minutes accumulates **litres**; a velocity in m/s over seconds accumulates **metres**.

Practice 2.1 ▪ 1 mark

A tap runs at a constant 2.5 L/min for 8 minutes. How much water flows out?

Solution 2.1

Method: Area under a rate graph is a total

Worked steps

$2.5 \times 8 = 20$ L **B1**.

Practice 2.2 ▪ 2 marks

A rate graph is a triangle: the rate rises linearly from 0 to 10 units/s over 4 s. Find the accumulated change.

Solution 2.2

Method: Splitting into simple shapes

Worked steps

Triangle area $= \frac{1}{2}(4)(10)$ M1 $= 20$ units A1.

Practice 2.3 ▪ 1 mark

Over one interval a quantity gains an area of 12 and over the next it loses an area of 5. State the net accumulated change.

Solution 2.3

Method: Signed area

Worked steps

$12 - 5 = 7$ B1.

Exam-style 3.1 ▪ 1 mark

The area under a velocity-vs-time graph represents

A acceleration

B displacement

C speed

D force

Solution 3.1

A acceleration

B displacement ✓

C speed

D force

Worked steps

B — the area under a velocity–time graph is displacement.

Exam-style 3.2 ▪ 1 mark

A machine consumes power at a rate $P(t)$ kilowatts, shown as a graph made of straight segments: a rectangle of height 2 over $0 \le t \le 3$ hours, then a triangle rising from 0 to 4 over $3 \le t \le 5$ hours.

(a) Find the energy used over $0 \le t \le 3$.

(b) Find the energy used over $3 \le t \le 5$.

(c) State the total energy used, with units.

Solution 3.2

Method: Splitting into simple shapes

Worked steps

(a) $2\times3 = 6$ kWh B1. (b) triangle $\frac{1}{2}(2)(4)$ M1 $= 4$ kWh A1. (c) $6+4 = 10$ kWh A1.

Exam-style 3.3 ▪ 1 mark

Sand is added to a pile at a rate that gives an accumulated area of 30 (kg) over the first hour, then removed at a rate giving an area of 8 (kg) over the next hour. State the net change in the pile's mass.

Solution 3.3

Method: Signed area

Worked steps

$30 - 8 = 22$ kg **B1**.