Past-paper walk-through

Connecting a Function, Its First Derivative, and Its Second Derivative ▪ 5.9

eXercise

Questions in this set

- 2018 Q3
- 2016 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2018 ▪ Q3

[Graph of g, a continuous piecewise curve on the x-axis from -5 to 6 and y-axis from -3 to 8: For $x < -3$ (extending left of -5), the graph is a horizontal segment at $y = -3$. From $x = -3$ to $x = -1$, the graph rises linearly from $(-3, -3)$ to $(-1, 0)$. From $x = -1$ to $x = 0$, the graph is a horizontal segment at $y = 0$. From $x = 0$ to about $x = 1$, the graph rises from $(0, 0)$ to $(1, 2)$. From $x = 1$ to $x = 3$, the graph is a horizontal segment at $y = 2$. From $x = 3$ to $x = 4$, the graph dips down to a minimum at $(4, -1)$ then rises back to $(5, 0)$, forming a smooth curve. From $x = 5$ to $x = 6$, the graph rises steeply through $(5, 0)$ up to $(6, 8)$. Labeled "Graph of g".]

The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \leq x < 3$, and $g(x) = 2(x-4)^2$ for $3 \leq x \leq 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

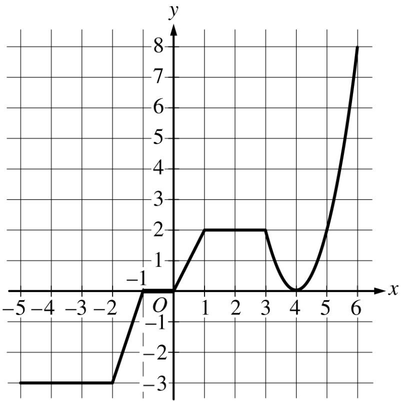
Question 3

(b) Evaluate $\int_1^6 g(x)\,dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

Question 3



Graph of g

Solution 3

(a) Mark scheme

- $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx = 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$. Points: 1 for the integral setup relating $f(-5)$ to $f(1)$ via the Fundamental Theorem of Calculus (using $f' = g$ on $[-5, 1]$), 1 for the correct answer $\frac{25}{2}$.

Solution 3

(b)

Mark scheme

- $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx = \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx = 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \left(\frac{16}{3} - \left(-\frac{2}{3}\right)\right) = 10$. Points: 1 for splitting the integral at $x = 3$ (piecewise definition of g), 1 for the correct antiderivative of $2(x-4)^2$, 1 for the correct answer (10).

Solution 3

(c)

Mark scheme

- The graph of f is increasing and concave up on $0 < x < 1$ and $4 < x < 6$, because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals. Points: 1 for identifying the correct intervals, 1 for the reason (both conditions $f' = g > 0$ and g increasing).

Solution 3

(d)

Mark scheme

- The graph of f has a point of inflection at $x = 4$, because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$. Points: 1 for the correct x-coordinate, 1 for the reason (concavity change identified via $f' = g$ changing from decreasing to increasing, i.e. g has a minimum there).

Question 3

2016 ▪ Q3

The figure above shows the graph of the piecewise-linear function f. For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_{2}^{x} f(t)\, dt$.

[Graph of f: piecewise-linear function on axes with x from -4 to 12 (gridlines every 2) and y from -4 to 4. Key labeled points: $(-4,-4)$, then rising to $(0,4)$, down to $(2,0)$, up to $(4,4)$, down to $(8,-4)$, up to $(10,0)$ [labeled 10 on the x-axis], down to $(12,-4)$.]

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

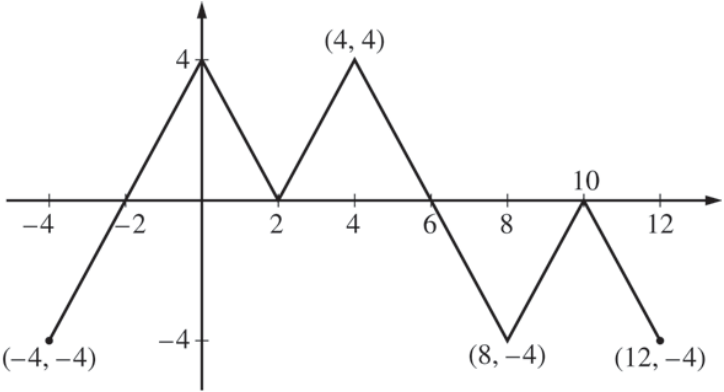
(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.

Question 3

(d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

Question 3



Graph of f

Solution 3

(a)

Mark scheme

- Since $g(x) = \int_2^x f(t)\,dt$, the Fundamental Theorem of Calculus gives $g'(x) = f(x)$ (this key relationship is worth 1 point, creditable in any part of the question). On $8 \leq x \leq 12$ the graph shows $f(x) \leq 0$, touching 0 only at $x = 10$, so $g'(x) = f(x)$ does not change sign at $x = 10$. Therefore g has NEITHER a relative minimum NOR a relative maximum at $x = 10$. Award 1 point for establishing $g'(x) = f(x)$, and 1 point for the correct answer with justification.

Solution 3

(b)

Mark scheme

- $g'(x) = f(x)$ is increasing for $2 \leq x \leq 4$ and decreasing for $4 \leq x \leq 8$ (read from the graph of f), so g is concave up then concave down at $x = 4$. So YES, the graph of g has a point of inflection at $x = 4$. Award 1 point for the answer with justification.

Solution 3

(c) Mark scheme

- $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$, so these are candidates for extrema; the endpoints $x = -4$ and $x = 12$ must also be considered. Evaluating g (the signed area under f from 2 to x, from the graph) at each candidate gives the table $g(-4) = -4$, $g(-2) = -8$, $g(6) = 8$, $g(12) = -4$. On $-4 \leq x \leq 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$. Award 1 point for considering $x = -2$ and $x = 6$ as candidates, 1 point for considering the endpoints $x = -4$ and $x = 12$ as candidates, and 2 points for the two correct answers with justification (citing the candidate-value table).

Solution 3

(d)

Mark scheme

- $g(x) \leq 0$ for $-4 \leq x \leq 2$ and for $10 \leq x \leq 12$, matching the sign of g determined from the candidate-value table (g is ≤ 0 from the left endpoint through $x = 2$, where $g(2) = 0$ by definition, and again from $x = 10$, where g returns to 0, through the right endpoint $x = 12$). Award 2 points for correctly identifying both intervals.