Exercise walk-through

Connecting a Function, Its First Derivative, and Its Second Derivative ▪ 5.9

eXercise

You must be able to

- read f, f', and f'' together to describe a graph

Method — Connecting f, f', f''

- f' tells increasing/decreasing;
- f'' tells concavity;
- a local **min** of $f \Leftrightarrow f' = 0$ and $f'' > 0$;
- an **inflection** point of $f \Leftrightarrow f''$ changes sign.

Method — Example

$f(x) = x^3$: $f'(x) = 3x^2$ (zero at 0), $f''(x) = 6x$ (zero at 0).

Practice 2.1 ▪ 1 mark

State what f'' tells you about f.

Solution 2.1

Worked steps

its concavity (concave up where $f'' > 0$, down where $f'' < 0$) B1.

Practice 2.2 ▪ 1 mark

State the condition on f' and f'' at a local minimum.

Solution 2.2

Worked steps

$f' = 0$ and $f'' > 0$ B1.

Practice 2.3 ▪ 2 marks

For $f(x) = x^3$, state where $f' = 0$ and where $f'' = 0$.

Solution 2.3

Worked steps

$f' = 0$ at $x = 0$; $f'' = 0$ at $x = 0$ B1 B1.

Exam-style 3.1 ▪ 1 mark

f' gives information about a graph's

A slope

B concavity

C x-intercepts

D domain

Solution 3.1

A slope

B concavity ✓

C x-intercepts

D domain

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

A local minimum of f satisfies

A $f' = 0$, $f'' > 0$

B $f' = 0$, $f'' < 0$

C $f'' = 0$ only

D $f = 0$

Solution 3.2

A $f = 0,\ f' > 0$ ✓

B $f = 0,\ f' < 0$

C $f' = 0$ only

D $f = 0$

Worked steps

A.

Exam-style 3.3 ▪ 1 mark

$f(x) = x^3$.
(a) Find $f'(x)$.
(b) State where $f' = 0$.
(c) State where $f'' = 0$.

Solution 3.3

Worked steps

(a) $3x^2$ B1. (b) $x = 0$ B1. (c) $x = 0$ B1.