Exercise walk-through

Determining Concavity of Functions over Their Domains ▪ 5.6

eXercise

You must be able to

- use the sign of f'' to find **concavity 凹凸性**
- identify an **inflection point 拐点**

Method — Concavity

- f is **concave up** where $f'' > 0$;
- f is **concave down** where $f'' < 0$.

An **inflection point** is where concavity changes (and f'' changes sign).

Method — Example

$f(x) = x^3$: $f''(x) = 6x$, which is > 0 for $x > 0$; the inflection point is at $x = 0$.

Practice 2.1 ▪ 1 mark

State where f is concave up.

Solution 2.1

Method: Concavity

Worked steps

where $f'' > 0$ **B1**.

Practice 2.2 ▪ 2 marks

For $f(x) = x^3$, find f'' and where it is concave up.

Solution 2.2

Method: Concavity

Worked steps

$f''(x) = 6x$; concave up for $x > 0$ **M1** **A1**.

Practice 2.3 ▪ 1 mark

State what an inflection point is.

Solution 2.3

Worked steps

a point where concavity changes (from up to down or vice versa) **B1**.

Exam-style 3.1 ▪ 1 mark

f is concave up where

A $f' > 0$

B $f'' > 0$

C $f'' < 0$

D $f = 0$

Solution 3.1

Method: Concavity

A $f' > 0$

B $f'' > 0$

C $f'' < 0$

D $f = 0$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

An inflection point is where

A $f' = 0$

B concavity changes

C $f = 0$

D f'' is constant

Solution 3.2

Method: Concavity

A $f' = 0$

B concavity changes ✓

C $f = 0$

D f'' is constant

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$f(x) = x^3$.
(a) Find $f''(x)$.
(b) State where $f'' > 0$.
(c) State the inflection point.

Solution 3.3

Method: Concavity

Worked steps

(a) $6x$ **B1**. (b) $x > 0$ **B1**. (c) $x = 0$ **B1**.