Exercise walk-through

Using the Mean Value Theorem ▪ 5.1

eXercise

You must be able to

- state the **Mean Value Theorem** 中値定理 and its hypotheses
- find the guaranteed point c

Method — The Mean Value Theorem

If f is continuous on $[a, b]$ and differentiable on (a, b), there is a c in (a, b) with

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

— somewhere the tangent slope equals the average slope.

Method — Example

$f(x) = x^2$ on $[0, 4]$: average slope $= \frac{16-0}{4} = 4$; $f'(c) = 2c = 4$ gives $c = 2$.

Practice 2.1 ▪ 1 mark

State the MVT conclusion.

Solution 2.1

Worked steps

there is a c in (a, b) where $f'(c)$ equals the average rate of change on $[a, b]$ B1.

Practice 2.2 ▪ 1 mark

State the two hypotheses.

Solution 2.2

Worked steps

f continuous on $[a, b]$ and differentiable on (a, b) B1.

Practice 2.3 ▪ 2 marks

For $f(x) = x^2$ on $[0, 4]$, find the average rate of change.

Solution 2.3

Worked steps

$$\frac{16-0}{4-0} = 4 \quad \text{M1}\ \text{A1}.$$

Exam-style 3.1 ▪ 1 mark

The MVT guarantees a c with $f'(c)$ equal to

A 0

B the average rate of change on $[a, b]$

C $f(a)$

D ∞

Solution 3.1

A 0

B the average rate of change on $[a, b]$ ✓

C $f(a)$

D ∞

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

The MVT requires f continuous on $[a, b]$ and

A $f = 0$

B differentiable on (a, b)

C linear

D increasing

Solution 3.2

A $f = 0$

B differentiable on (a, b) ✓

C linear

D increasing

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$f(x) = x^2$ on $[0, 4]$.
(a) Find $f(0)$ and $f(4)$.
(b) Find the average slope.
(c) Solve $f'(c) =$ that for c.

Solution 3.3

Worked steps

(a) $f(0) = 0$, $f(4) = 16$ B1. (b) 4 B1. (c) $2c = 4$, so $c = 2$ B1.