

Exercise walk-through

Using L'Hospital's Rule for Determining Limits of Indeterminate Forms ■ 4.7

eXercise

You must be able to

- recognise an **indeterminate form** 未定式 $\frac{0}{0}$ or $\frac{\infty}{\infty}$
- apply **L'Hospital's Rule** 洛必达法则

Method — L'Hospital's Rule

For a limit of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$,

$$\lim \frac{f}{g} = \lim \frac{f'}{g'}$$

(when the right-hand limit exists).

Method — Example

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = 1.$$

Practice 2.1 ■ 1 mark

State the forms L'Hospital's Rule applies to.

Solution 2.1

Method: L'Hospital's Rule

Worked steps

$\frac{0}{0}$ and $\frac{\infty}{\infty}$ **B1**.

Practice 2.2 ■ 2 marks

Apply it to $\lim_{x \rightarrow 0} \frac{\sin x}{x}$.

Solution 2.2

Worked steps

$$\lim_{x \rightarrow 0} \frac{\cos x}{1} = 1 \quad \text{M1} \quad \text{A1}.$$

Practice 2.3 ■ 1 mark

State what you do to the numerator and denominator.

Solution 2.3

Worked steps

differentiate each separately, then take the limit of the new quotient **B1**.

Exam-style 3.1 ■ 1 mark

L'Hospital's Rule applies to

A any limit

B $\frac{0}{0}$ or $\frac{\infty}{\infty}$

C $\frac{1}{0}$

D $\frac{5}{2}$

Solution 3.1

Method: L'Hospital's Rule

A any limit

B $\frac{0}{0}$ or $\frac{\infty}{\infty}$



C $\frac{1}{0}$

D $\frac{5}{2}$

Worked steps

B.

Exam-style 3.2 ■ 1 mark

L'Hospital's Rule replaces $\frac{f}{g}$ with

A $f \cdot g$

B $\frac{f'}{g'}$

C $\frac{f'}{g}$

D $\frac{f}{g'}$

Solution 3.2

Method: L'Hospital's Rule

A $f \cdot g$

B $\frac{f'}{g'}$



C $\frac{f'}{g}$

D $\frac{f}{g'}$

Worked steps

B.

Exam-style 3.3 ■ 1 mark

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x}.$$

- (a) Check the form.
- (b) Differentiate top and bottom.
- (c) Evaluate.

Solution 3.3

Worked steps

(a) $\frac{0}{0}$ B1. (b) $\frac{e^x}{1}$ B1. (c) 1 B1.