Exercise walk-through

Approximating Values of a Function Using Local Linearity and Linearization ▪ 4.6

eXercise

You must be able to

- write the **linearization 线性化** $L(x) = f(a) + f'(a)(x - a)$
- use it to approximate a value

Method — Linearization

Near $x = a$,

$$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

— the tangent line at a.

Method — Example

$f(x) = \sqrt{x}$ at $a = 4$: $f(4) = 2$, $f'(4) = \frac{1}{4}$, so $L(x) = 2 + \frac{1}{4}(x - 4)$ and $\sqrt{4.1} \approx 2.025$.

Practice 2.1 ▪ 1 mark

Write the linearization formula.

Solution 2.1

Worked steps

$L(x) = f(a) + f'(a)(x - a)$ **B1**.

Practice 2.2 ▪ 2 marks

For $f(x) = \sqrt{x}$ at $a = 4$, state $f(4)$ and $f'(4)$.

Solution 2.2

Worked steps

$f(4) = 2$; $f'(4) = \frac{1}{4}$ **B1** **B1**.

Practice 2.3 ▪ 1 mark

State what $L(x)$ approximates.

Solution 2.3

Worked steps

the value of $f(x)$ for x near a B1.

Exam-style 3.1 ▪ 1 mark

The linearization of f at a is

A $f(a)$

B $f(a) + f'(a)(x - a)$

C $f'(a)(x - a)$

D $f(x) \cdot a$

Solution 3.1

A $f(a)$

B $f(a) + f'(a)(x - a)$ ✓

C $f'(a)(x - a)$

D $f(x) \cdot a$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

Linearization uses the ________ line.

- **A** secant
- **B** tangent
- **C** normal
- **D** horizontal

Solution 3.2

Method: Linearization

A secant

B tangent ✓

C normal

D horizontal

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$f(x) = x^2$, $a = 3$, so $f(3) = 9$, $f'(3) = 6$.
(a) Write $L(x)$.
(b) Use it for $x = 3.1$.
(c) State $L(3.1)$.

Solution 3.3

Worked steps

(a) $L(x) = 9 + 6(x - 3)$ **B1**. (b) $9 + 6(0.1)$ **B1**. (c) 9.6 **B1**.