

Exercise walk-through

Differentiating Inverse Functions ■ 3.3

eXercise

You must be able to

- apply $(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$

Method — Derivative of an inverse

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}.$$

The slope of the **inverse function** 反函数 is the reciprocal of the slope of f at the corresponding point.

Method — Example

If $f(2) = 5$ and $f'(2) = 4$, then $(f^{-1})'(5) = \frac{1}{f'(2)} = \frac{1}{4}$.

Practice 2.1 ■ 1 mark

State the inverse-derivative formula.

Solution 2.1

Method: Derivative of an inverse

Worked steps

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))} \quad \text{B1}.$$

Practice 2.2 ■ 2 marks

If $f'(2) = 4$ and $f(2) = 5$, find $(f^{-1})'(5)$.

Solution 2.2

Worked steps

$$f^{-1}(5) = 2, \text{ so } (f^{-1})'(5) = \frac{1}{f'(2)} = \frac{1}{4} \quad \text{M1 A1}.$$

Practice 2.3 ■ 1 mark

State the relationship between the two slopes.

Solution 2.3

Worked steps

they are reciprocals of each other **B1**.

Exam-style 3.1 ■ 1 mark

$(f^{-1})'(a)$ equals

A $f'(a)$

B $\frac{1}{f'(f^{-1}(a))}$

C $f'(f^{-1}(a))$

D $\frac{1}{a}$

Solution 3.1

A $f'(a)$

B $\frac{1}{f'(f^{-1}(a))}$



C $f'(f^{-1}(a))$

D $\frac{1}{a}$

Worked steps

B.

Exam-style 3.2 ■ 1 mark

The slope of f^{-1} is the _____ of the slope of f .

A same

B reciprocal

C negative

D square

Solution 3.2

A same

B reciprocal 

C negative

D square

Worked steps

B.

Exam-style 3.3 ■ 1 mark

$$f(3) = 7, f'(3) = 2.$$

- (a) Find $f^{-1}(7)$.
- (b) Apply the formula.
- (c) Find $(f^{-1})'(7)$.

Solution 3.3

Worked steps

(a) 3 **B1**. (b) $\frac{1}{f(3)}$ **B1**. (c) $\frac{1}{2}$ **B1**.