Exercise walk-through

Implicit Differentiation ▪ 3.2

eXercise

You must be able to

- differentiate an **implicit relation** 隐式关系 by attaching $\frac{dy}{dx}$ to every y
- solve for $\frac{dy}{dx}$

Method — Implicit differentiation

Differentiate both sides with respect to x, writing y' each time you differentiate y (the chain rule), then solve for y'.

Method — Example

$$x^2 + y^2 = 25 \;\Rightarrow\; 2x + 2y\,y' = 0 \;\Rightarrow\; y' = -\frac{x}{y}.$$

Practice 2.1 ▪ 1 mark

State what to attach when differentiating y.

Solution 2.1

Worked steps

the factor $\frac{dy}{dx}$ (i.e. y') B1.

Practice 2.2 ▪ 2 marks

Differentiate $x^2 + y^2 = 25$ implicitly.

Solution 2.2

Worked steps

$2x + 2y\,y' = 0$, so $y' = -\dfrac{x}{y}$ M1 A1.

Practice 2.3 ▪ 1 mark

State the final step.

Solution 2.3

Worked steps

solve the equation for $\frac{dy}{dx}$ B1.

Exam-style 3.1 ▪ 1 mark

In implicit differentiation, differentiating y^2 gives

A $2y$

B $2yy'$

C y'

D $2x$

Solution 3.1

Method: Implicit differentiation

A $2y$

B $2y\,y'$ ✓

C y'

D $2x$

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

After differentiating, you

A stop

B solve for $\frac{dy}{dx}$

C set $y = 0$

D integrate

Solution 3.2

A stop

B solve for $\frac{dy}{dx}$ ✓

C set $y = 0$

D integrate

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$x^2 + y^2 = 25$.

(a) Differentiate both sides.

(b) Isolate the y' term.

(c) Solve for y'.

Solution 3.3

Method: Implicit differentiation

Worked steps

(a) $2x + 2yy' = 0$ B1. (b) $2yy' = -2x$ B1. (c) $y' = -\dfrac{x}{y}$ B1.