Exercise walk-through

Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist ▪ 2.4

eXercise

You must be able to

- use that **differentiability 可导性** implies **continuity 连续性** (but not the reverse)
- identify where a continuous function fails to be differentiable

Method — Differentiability and continuity

If f is differentiable at a, then f is continuous at a. The converse **fails**: f can be continuous yet not differentiable — at a **corner**, cusp, or vertical tangent.

Method — Example

$f(x) = |x|$ is continuous at 0 but has a corner there, so it is not differentiable at 0.

Practice 2.1 ▪ 1 mark

State the implication between differentiability and continuity.

Solution 2.1

Method: Differentiability and continuity

Worked steps

differentiable at $a \Rightarrow$ continuous at a (B1).

Practice 2.2 ▪ 1 mark

State whether continuity implies differentiability.

Solution 2.2

Method: Differentiability and continuity

Worked steps

no (continuity does not imply differentiability) B1.

Practice 2.3 ▪ 2 marks

Give an example of a continuous but non-differentiable point.

Solution 2.3

Method: Example

Worked steps

$f(x) = |x|$ at $x = 0$ (a corner) B1 B1.

Exam-style 3.1 ▪ 1 mark

If f is differentiable at a, then f is

A discontinuous

B continuous at a

C linear

D constant

Solution 3.1

Method: Differentiability and continuity

A discontinuous

B continuous at a ✓

C linear

D constant

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

$f(x) = |x|$ at $x = 0$ is

A differentiable

B continuous but not differentiable

C discontinuous

D undefined

Solution 3.2

A differentiable

B continuous but not differentiable ✓

C discontinuous

D undefined

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

Continuous functions can fail to be differentiable at certain features.
(a) Name the feature at a sharp point.
(b) Name the feature with an infinite slope.
(c) State whether $|x|$ is differentiable at 0.

Solution 3.3

Worked steps

(a) a corner B1. (b) a vertical tangent B1. (c) no B1.