Exercise walk-through

Estimating Derivatives of a Function at a Point ▪ 2.3

eXercise

You must be able to

- estimate $f'(a)$ from a **table** 表格 of values using a small-h difference quotient

Method — Estimating a derivative

Estimate $f'(a)$ with a small-h difference quotient from table values:

$$f'(a) \approx \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad \frac{f(a+h) - f(a-h)}{2h} \ \text{(symmetric)}.$$

Method — Example

$$f(2) = 5,\ f(2.1) = 5.4\text{:}\ \ f'(2) \approx \frac{5.4 - 5}{0.1} = 4.$$

Practice 2.1 ▪ 1 mark

State a formula to estimate $f'(a)$ from a table.

Solution 2.1

Method: Estimating a derivative

Worked steps

$f'(a) \approx \dfrac{f(a+h) - f(a)}{h}$ for small h B1.

Practice 2.2 ▪ 2 marks

If $f(2) = 5$ and $f(2.1) = 5.4$, estimate $f'(2)$.

Solution 2.2

Worked steps

$$\frac{5.4-5}{2.1-2}=\frac{0.4}{0.1}=4$$ M1 A1.

Practice 2.3 ▪ 1 mark

State why a smaller h generally gives a better estimate.

Solution 2.3

Worked steps

a smaller interval is closer to the tangent slope (the limit) B1.

Exam-style 3.1 ▪ 1 mark

A derivative at a point can be estimated from a table using a

A single value

B difference quotient with small h

C maximum

D mean

Solution 3.1

Method: Estimating a derivative

A single value

B difference quotient with small h ✓

C maximum

D mean

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

The symmetric difference quotient is

A $\dfrac{f(a+h)-f(a)}{h}$

B $\dfrac{f(a+h)-f(a-h)}{2h}$

C $\dfrac{f(a)}{h}$

D $f(a+h)\cdot f(a-h)$

Solution 3.2

Method: Estimating a derivative

A $\dfrac{f(a+h)-f(a)}{h}$

B $\dfrac{f(a+h)-f(a-h)}{2h}$ ✓

C $\dfrac{f(a)}{h}$

D $f(a+h)\cdot f(a-h)$

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$f(3) = 10$, $f(3.2) = 10.6$.
(a) Find Δf.
(b) Find Δx.
(c) Estimate $f'(3)$.

Solution 3.3

Worked steps

(a) 0.6 B1. (b) 0.2 B1. (c) $\dfrac{0.6}{0.2} = 3$ B1.