

## Exercise walk-through

Defining Average and Instantaneous Rates of Change at a Point ■ 2.1

eXercise

You must be able to

- form the **difference quotient** 差商 for the average rate of change
- describe the instantaneous rate as its limit

## Method — Average and instantaneous rates

Average rate over  $[a, a + h]$  is the **secant slope**  $\frac{f(a + h) - f(a)}{h}$ .

The instantaneous rate at  $a$  is the **tangent slope**, obtained as  $h \rightarrow 0$ .

## Method — Example

$$f(x) = x^2 \text{ at } a = 3: \frac{(3+h)^2 - 9}{h} = 6 + h \rightarrow 6 \text{ as } h \rightarrow 0.$$

## Practice 2.1 ■ 1 mark

State the difference quotient for the average rate over  $[a, a + h]$ .

## Solution 2.1

Method: Average and instantaneous rates

### Worked steps

$$\frac{f(a+h) - f(a)}{h} \quad \text{B1}.$$

## Practice 2.2 ■ 1 mark

State what happens to it as  $h \rightarrow 0$ .

## Solution 2.2

### Worked steps

it approaches the instantaneous rate of change (the derivative) **B1**.



## Practice 2.3 ■ 2 marks

For  $f(x) = x^2$ , compute  $\frac{f(2+h) - f(2)}{h}$  and simplify.

## Solution 2.3

## Worked steps

$$\frac{(2+h)^2 - 4}{h} = \frac{4h + h^2}{h} = 4 + h \quad \text{M1} \quad \text{A1}.$$

## Exam-style 3.1 ■ 1 mark

The average rate of change over  $[a, a + h]$  is

**A**  $f(a)h$

**B**  $\frac{f(a + h) - f(a)}{h}$

**C**  $f'(a)$

**D**  $h$

## Solution 3.1

Method: Average and instantaneous rates

A  $f(a)h$

B  $\frac{f(a+h) - f(a)}{h}$



C  $f'(a)$

D  $h$

### Worked steps

B.

## Exam-style 3.2 ■ 1 mark

The instantaneous rate is the average rate as

**A**  $h \rightarrow \infty$

**B**  $h \rightarrow 0$

**C**  $h = 1$

**D**  $a \rightarrow 0$

## Solution 3.2

Method: Average and instantaneous rates

**A**  $h \rightarrow \infty$

**B**  $h \rightarrow 0$



**C**  $h = 1$

**D**  $a \rightarrow 0$

### Worked steps

**B.**

## Exam-style 3.3 ■ 1 mark

$$f(x) = x^2 \text{ at } a = 3.$$

(a) Write  $f(3 + h)$ .

(b) Form the difference quotient.

(c) State its limit as  $h \rightarrow 0$ .

## Solution 3.3

## Worked steps

$$(a) (3 + h)^2 \quad \text{B1}. \quad (b) \frac{(3 + h)^2 - 9}{h} = 6 + h \quad \text{B1}. \quad (c) 6 \quad \text{B1}.$$