Exercise walk-through

Determining Absolute or Conditional Convergence ▪ 10.9

eXercise

You must be able to

- test whether $\sum |a_n|$ converges (**absolute convergence 绝对收敛**)
- classify a convergent series as absolutely or **conditionally convergent 条件收敛**
- use the standard example $\sum \frac{(-1)^n}{n}$

Method — Absolute convergence

$\sum a_n$ **converges absolutely** if $\sum |a_n|$ converges. Absolute convergence is the **stronger** property — it implies the series itself converges.

Method — Conditional convergence

If $\sum a_n$ converges but $\sum |a_n|$ diverges, the series **converges conditionally** — it relies on the cancellation of signs.

Method — The standard example

$\sum \dfrac{(-1)^n}{n}$ converges (alternating series test), but $\sum \left|\dfrac{(-1)^n}{n}\right| = \sum \dfrac{1}{n}$ diverges. So it is **conditionally** convergent.

Method — The routine

1. Test $\sum |a_n|$. If it converges $\to$ **absolute**. 2. If not, test $\sum a_n$ itself (e.g. alternating series). If it converges $\to$ **conditional**; if not $\to$ **divergent**.

Practice 2.1 ▪ 1 mark

For $\sum \frac{(-1)^n}{n^2}$, does $\sum \left|\frac{(-1)^n}{n^2}\right|$ converge?

Solution 2.1

Method: The standard example

Worked steps

Yes — $\sum \frac{1}{n^2}$ is a convergent p-series **B1**.

Practice 2.2 ▪ 1 mark

Classify $\sum \frac{(-1)^n}{n^2}$ (absolute / conditional).

Solution 2.2

Worked steps

Absolutely convergent **B1**.

Practice 2.3 ▪ 2 marks

For $\sum \frac{(-1)^n}{n}$, does $\sum \frac{1}{n}$ converge? What does that make the series?

Solution 2.3

Method: The standard example

Worked steps

$\sum \frac{1}{n}$ diverges **B1**; so $\sum \frac{(-1)^n}{n}$ is **conditionally** convergent **B1**.

Exam-style 3.1 ▪ 1 mark

A series is **conditionally** convergent if

A both $\sum a_n$ and $\sum |a_n|$ converge

B $\sum a_n$ converges but $\sum |a_n|$ diverges

C both diverge

D $\sum |a_n|$ converges but $\sum a_n$ diverges

Solution 3.1

Method: The standard example

A both $\sum a_n$ and $\sum |a_n|$ converge

B $\sum a_n$ converges but $\sum |a_n|$ diverges ✓

C both diverge

D $\sum |a_n|$ converges but $\sum a_n$ diverges

Worked steps

B — converges, but not absolutely.

Exam-style 3.2 ▪ 2 marks

Consider $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(a) Test $\displaystyle\sum \frac{1}{\sqrt{n}}$ for convergence.

(b) Test the alternating series itself.

(c) Classify the series.

Solution 3.2

Method: The standard example

Worked steps

(a) $\sum \frac{1}{\sqrt{n}}$ is a p-series with $p = \frac{1}{2} \leq 1$, diverges **M1** **A1**. (b) Alternating series test: $b_n = \frac{1}{\sqrt{n}}$ decreasing $\to 0$, so it converges **B1**. (c) Conditionally convergent **B1**.

Exam-style 3.3 ▪ 2 marks

Explain why absolute convergence is “stronger” than conditional convergence.

Solution 3.3

Method: Absolute convergence

Worked steps

Absolute convergence guarantees the series converges no matter how the terms are arranged, and it implies ordinary convergence; conditional convergence depends on the sign pattern and can be broken by rearrangement **M1** **A1**.