Exercise walk-through

Alternating Series Test for Convergence ▪ 10.7

eXercise

You must be able to

- recognise an **alternating series 交错级数** $\sum(-1)^n b_n$
- check the three conditions ($b_n > 0$, decreasing, $\to 0$)
- conclude convergence when they hold

Method — The alternating series test

An alternating series $\sum(-1)^n b_n$ (with $b_n > 0$) **converges** if:

1. b_n is **decreasing** $(b_{n+1} \leq b_n)$, and
2. $\lim_{n\to\infty} b_n = 0$.

Method — A worked example

$\sum \frac{(-1)^n}{n}$: $b_n = \frac{1}{n}$ is positive, decreasing, and $\to 0$. So the **alternating harmonic series converges** — even though $\sum \frac{1}{n}$ diverges.

Method — Both conditions matter

If the terms b_n do **not** go to 0, the nth term test already forces divergence. If $b_n \to 0$ but is not decreasing, the test does not directly apply.

Method — Recognising the form

The sign alternates via $(-1)^n$ or $(-1)^{n+1}$. Peel off the sign to get b_n, then test b_n.

Practice 2.1 ▪ 1 mark

For $\sum \frac{(-1)^n}{n^2}$, identify b_n.

Solution 2.1

Method: A worked example

Worked steps

$b_n = \frac{1}{n^2}$ **B1**.

Practice 2.2 ▪ 2 marks

Check the two conditions for $\sum \frac{(-1)^n}{n^2}$.

Solution 2.2

Method: A worked example

Worked steps

$b_n = \frac{1}{n^2} > 0$ and decreasing; $\lim b_n = 0$ **M1** **A1**.

Practice 2.3 ▪ 1 mark

State whether $\sum \frac{(-1)^n}{n^2}$ converges.

Solution 2.3

Method: A worked example

Worked steps

Converges B1.

Exam-style 3.1 ▪ 1 mark

The alternating series test requires that b_n is positive, tends to 0, and is

- **A** increasing
- **B** decreasing
- **C** constant
- **D** geometric

Solution 3.1

- **A** increasing
- **B** decreasing ✓
- **C** constant
- **D** geometric

Worked steps

B — b_n must be decreasing.

Exam-style 3.2 ▪ 3 marks

Consider $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$.

(a) Identify b_n and check the conditions.

(b) State the conclusion.

Solution 3.2

Worked steps

(a) $b_n = \frac{1}{\sqrt{n}} > 0$, decreasing, and $\to 0$ **M1** **M1** **A1**. (b) Converges by the alternating series test **B1**.

Exam-style 3.3 ▪ 2 marks

Explain why the alternating harmonic series $\sum \frac{(-1)^n}{n}$ converges even though the harmonic series diverges.

Solution 3.3

Method: A worked example

Worked steps

The alternating signs let successive terms partly cancel, so the partial sums settle to a limit; without the signs ($\sum \frac{1}{n}$) there is no cancellation and the sum grows without bound **M1** **A1**.