Exercise walk-through

The nth Term Test for Divergence ▪ 10.3

eXercise

You must be able to

- apply: if $\lim_{n \to \infty} a_n \neq 0$, the series **diverges**
- recognise that the test is only a test for **divergence**
- know that $a_n \to 0$ does **not** prove convergence

Method — The test

If the terms do not shrink to zero, the sum cannot settle:

$$\lim_{n\to\infty} a_n \neq 0 \;\Rightarrow\; \sum a_n \text{ diverges.}$$

Method — A divergent example

$\sum \frac{n}{n+1}$: $\frac{n}{n+1} \to 1 \neq 0$, so the series **diverges** by the *n*th term test.

Method — It only shows divergence

If $\lim a_n = 0$, the test is **inconclusive** — the series may converge or diverge. The harmonic series $\sum \frac{1}{n}$ has terms $\to 0$ yet **diverges**.

Method — First check to do

Because it is quick, always apply the nth term test **first**. Only if the limit is 0 do you move on to another test.

Practice 2.1 ▪ 2 marks

Find $\lim_{n \to \infty} \frac{2n}{3n+1}$.

Solution 2.1

Method: A divergent example

Worked steps

$$\frac{2n}{3n+1} \to \frac{2}{3} \quad \textbf{M1}\ \textbf{A1}.$$

Practice 2.2 ▪ 2 marks

State whether $\sum \frac{2n}{3n+1}$ converges or diverges, with a reason.

Solution 2.2

Method: A divergent example

Worked steps

Diverges — the limit is $\frac{2}{3} \neq 0$, so by the *n*th term test it diverges **M1** **A1**.

Practice 2.3 ▪ 1 mark

For $\sum \frac{1}{n}$, the terms go to 0. Does the *n*th term test prove convergence?

Solution 2.3

Method: A divergent example

Worked steps

No — the test is inconclusive when the terms go to 0 **B1**.

Exam-style 3.1 ▪ 1 mark

If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$

A converges

B diverges

C may converge or diverge (test inconclusive)

D equals 0

Solution 3.1

A converges

B diverges

C may converge or diverge (test inconclusive) ✓

D equals 0

Worked steps

C — the test is inconclusive when $a_n \to 0$.

Exam-style 3.2 ▪ 2 marks

Consider $\sum_{n=1}^{\infty} \frac{n^2}{2n^2+5}$.

(a) Find $\lim_{n\to\infty} a_n$.

(b) State the conclusion of the nth term test.

Solution 3.2

Method: A divergent example

Worked steps

(a) $\dfrac{n^2}{2n^2+5} \to \dfrac{1}{2}$ M1 A1. (b) Limit $\neq 0$, so the series **diverges** M1 A1.

Exam-style 3.3 ▪ 2 marks

A student claims that because the terms of $\sum \frac{1}{\sqrt{n}}$ tend to 0, the series converges. Explain the error.

Solution 3.3

Worked steps

The nth term test only detects **divergence**; $a_n \to 0$ does not prove convergence (in fact $\sum \frac{1}{\sqrt{n}}$ is a p-series with $p = \frac{1}{2} \leq 1$ and diverges) **M1** **A1**.