Exercise walk-through

Working with Geometric Series ▪ 10.2

eXercise

You must be able to

- identify the first term a and common **ratio** 比値 r
- apply the convergence test $|r| < 1$
- compute the sum $\dfrac{a}{1-r}$

Method — The geometric sum

A geometric series $\sum_{n=0}^{\infty} ar^n$ converges **exactly when** $|r| < 1$, and then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

Method — Reading a and r

For $\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n$: $a = 3$, $r = \frac{1}{2}$. Since $|r| < 1$, sum $= \dfrac{3}{1 - \frac{1}{2}} = 6.$

Method — Watch the starting index

If the sum starts at $n = 1$, the first term is ar, not a. $\displaystyle\sum_{n=1}^{\infty} \left(\tfrac{1}{3}\right)^n = \frac{1/3}{1 - 1/3} = \frac{1/3}{2/3} = \tfrac{1}{2}.$

Method — Divergence

If $|r| \geq 1$ the terms do not shrink, so the series diverges. $\sum 2^n$ has $r = 2$, diverges.

Practice 2.1 ▪ 2 marks

State a and r for $\sum_{n=0}^{\infty} 5\left(\frac{1}{3}\right)^n$.

Solution 2.1

Worked steps

$a = 5$, $r = \frac{1}{3}$ B1 B1.

Practice 2.2 ▪ 2 marks

Find the sum of $\sum_{n=0}^{\infty} 5\left(\frac{1}{3}\right)^n$.

Solution 2.2

Worked steps

$$\frac{5}{1-1/3} = \frac{5}{2/3} = \frac{15}{2}$$ M1 A1.

Practice 2.3 ▪ 1 mark

Does $\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$ converge? Give a reason.

Solution 2.3

Worked steps

No — $|r| = \frac{5}{4} > 1$, so it diverges B1.

Exam-style 3.1 ▪ 1 mark

A geometric series $\sum ar^n$ converges when

A $r > 0$

B $|r| < 1$

C $|r| > 1$

D $a < 1$

Solution 3.1

Method: The geometric sum

A $r > 0$

B $|r| < 1$ ✓

C $|r| > 1$

D $a < 1$

Worked steps

B — converges iff $|r| < 1$.

Exam-style 3.2 ▪ 2 marks

Consider $\sum_{n=0}^{\infty} 6\left(-\frac{1}{2}\right)^n$.

(a) State a and r.

(b) Find the sum.

Solution 3.2

Worked steps

(a) $a = 6$, $r = -\frac{1}{2}$ B1 B1. (b) $\dfrac{6}{1-(-\frac{1}{2})} = \dfrac{6}{3/2} = 4$ M1 A1.

Exam-style 3.3 ▪ 3 marks

Find the sum of $\sum_{n=1}^{\infty} 4\left(\frac{2}{5}\right)^n$ (note the index starts at 1).

Solution 3.3

Method: Watch the starting index

Worked steps

First term $(n = 1)$ is $4 \cdot \frac{2}{5} = \frac{8}{5}$, ratio $\frac{2}{5}$ M1; sum $= \dfrac{8/5}{1 - 2/5} = \dfrac{8/5}{3/5} = \frac{8}{3}$ M1 A1.