Exercise walk-through

Radius and Interval of Convergence of Power Series ▪ 10.13

eXercise

You must be able to

- use the **ratio test** to find the **radius of convergence 收敛半径** R
- test the **endpoints** separately
- state the full **interval of convergence 收敛区间**

Method — Radius by the ratio test

A power series $\sum c_n(x-a)^n$ converges for $|x-a|<R$. Find R by applying the ratio test and requiring $L<1$.

Method — A worked radius

$\sum \frac{(x-2)^n}{3^n}$: $\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x-2|}{3} \to \frac{|x-2|}{3}$. Set < 1: $|x-2| < 3$, so $R = 3$, center 2: the open interval is $(-1, 5)$.

Method — Test the endpoints

The ratio test is inconclusive at $x = -1$ and $x = 5$. At $x = 5$: $\sum 1$ diverges; at $x = -1$: $\sum(-1)^n$ diverges. So the interval is the open $(-1, 5)$.

Method — Stating the interval

Combine the open interval with each endpoint's separate result, using [ ] (included) or ( ) (excluded). Always test **both** ends.

Practice 2.1 ▪ 3 marks

For $\sum \frac{x^n}{2^n}$, apply the ratio test to find R.

Solution 2.1

Method: Radius by the ratio test

Worked steps

$\left|\dfrac{x^{n+1}/2^{n+1}}{x^n/2^n}\right| = \dfrac{|x|}{2}$ **M1**; set < 1: $|x| < 2$ **M1**; $R = 2$ **A1**.

Practice 2.2 ▪ 1 mark

State the open interval of convergence for a series centered at 0 with $R = 2$.

Solution 2.2

Worked steps

$(-2, 2)$ **B1**.

Practice 2.3 ▪ 1 mark

Why must the endpoints be tested separately?

Solution 2.3

Worked steps

The ratio test gives $L = 1$ at the endpoints, so it is inconclusive there **B1**.

Exam-style 3.1 ▪ 1 mark

The radius of convergence is found using the

A integral test

B ratio test

C nth term test

D comparison test

Solution 3.1

A integral test

B ratio test ✓

C nth term test

D comparison test

Worked steps

B — the ratio test gives the radius.

Exam-style 3.2 ▪ 3 marks

Consider $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$.

(a) Find the radius of convergence.

(b) Test the endpoints $x = 0$ and $x = 2$.

(c) State the interval of convergence.

Solution 3.2

Method: A worked radius

Worked steps

(a) $\left|\dfrac{(x-1)^{n+1}/(n+1)}{(x-1)^n/n}\right| = |x-1| \cdot \dfrac{n}{n+1} \to |x-1|; < 1 \Rightarrow R = 1$ **M1** **M1** **A1**. (b) $x = 2$: $\sum \frac{1}{n}$ diverges; $x = 0$: $\sum \frac{(-1)^n}{n}$ converges **M1** **A1**. (c) $[0, 2)$ **B1**.

Exam-style 3.3 ▪ 3 marks

Find the radius of convergence of $\sum \frac{x^n}{n!}$.

Solution 3.3

Method: A worked radius

Worked steps

$\left|\frac{x^{n+1}/(n+1)!}{x^n/n!}\right| = \frac{|x|}{n+1} \to 0 < 1$ for all x **M1** **M1**; $R = \infty$ (converges everywhere) **A1**.