# Exercise walk-through

## Finding Taylor Polynomial Approximations of Functions ▪ 10.11

eXercise

# You must be able to

- build a **Taylor polynomial 泰勒多项式** from derivative values at a center
- use the formula $P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$
- estimate a function value with the polynomial

# Method — The Taylor polynomial

Centered at $x = a$:

$$P_n(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x - a)^n.$$

Centered at $a = 0$ it is a **Maclaurin polynomial**.

# Method — Building from a table

Given $f(0) = 2$, $f'(0) = 3$, $f''(0) = -4$, the 2nd-degree Maclaurin polynomial is

$$P_2(x) = 2 + 3x + \frac{-4}{2!}x^2 = 2 + 3x - 2x^2.$$

# Method — A worked polynomial for e^x

For $f(x) = e^x$ at $a = 0$: all derivatives are $1$, so $P_3(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$.

# Method — Estimating a value

Use $P_n$ near the center. $e^{0.1} \approx P_3(0.1) = 1 + 0.1 + 0.005 + 0.000167 \approx 1.10517$.

# Practice 2.1 ▪ 1 mark

Write the general term $\dfrac{f^{(k)}(a)}{k!}(x-a)^k$ for $k = 2$.

# Solution 2.1

## Worked steps

$$\frac{f''(a)}{2!}(x-a)^2 \quad \boxed{\textbf{B1}}.$$

# Practice 2.2 ▪ 2 marks

Given $f(0) = 1$, $f'(0) = 2$, $f''(0) = 6$, write $P_2(x)$ (Maclaurin).

# Solution 2.2

Method: Building from a table

**Worked steps**

$P_2(x) = 1 + 2x + \frac{6}{2}x^2 = 1 + 2x + 3x^2$ M1 A1.

# Practice 2.3 ▪ 2 marks

For $f(x) = e^x$, write the 2nd-degree Maclaurin polynomial.

# Solution 2.3

Method: Building from a table

## Worked steps

$1 + x + \frac{x^2}{2}$ **M1** **A1**.

# Exam-style 3.1 ▪ 1 mark

The coefficient of $(x-a)^2$ in a Taylor polynomial is

**A** $f''(a)$

**B** $\dfrac{f''(a)}{2}$

**C** $2f''(a)$

**D** $f'(a)$

# Solution 3.1

**Method:** The Taylor polynomial

**A** $f''(a)$

**B** $\dfrac{f''(a)}{2}$ ✓

**C** $2f''(a)$

**D** $f'(a)$

## Worked steps

**B** — the coefficient is $\dfrac{f''(a)}{2!} = \dfrac{f''(a)}{2}$.

# Exam-style 3.2 ▪ 3 marks

A function has $f(1) = 3$, $f'(1) = -2$, $f''(1) = 4$, $f'''(1) = 12$.
(a) Write the 3rd-degree Taylor polynomial about $x = 1$.
(b) Use it to estimate $f(1.1)$.

# Solution 3.2

## Worked steps

(a) $P_3(x) = 3-2(x-1)+\frac{4}{2}(x-1)^2+\frac{12}{6}(x-1)^3 = 3-2(x-1)+2(x-1)^2+2(x-1)^3$ **M1** **M1** **A1**. (b) at $x = 1.1$: $3 - 0.2 + 2(0.01) + 2(0.001) = 2.822$ **M1** **A1**.

# Exam-style 3.3 ▪ 3 marks

Write the 4th-degree Maclaurin polynomial for $f(x) = \cos x$ (using $\cos 0 = 1$, and the derivative pattern).

# Solution 3.3

Method: Building from a table

## Worked steps

Derivatives of $\cos$ at $0$: $1, 0, -1, 0, 1$; $P_4(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$ M1 M1 A1.