Exercise walk-through

Alternating Series Error Bound ▪ 10.10

eXercise

You must be able to

- bound the error of a partial sum by the **first omitted term** b_{N+1}
- find how many terms guarantee a required accuracy
- apply $|S - S_N| \leq b_{N+1}$ correctly

Method — The error bound

For a **convergent alternating series**, the error in stopping at S_N is no larger than the **first term you left out**:

$$|S - S_N| \leq b_{N+1}.$$

Method — A worked bound

$\sum \frac{(-1)^{n+1}}{n}$: using $S_3 = 1 - \frac{1}{2} + \frac{1}{3}$, the error is at most $b_4 = \frac{1}{4} = 0.25$.

Method — How many terms for a target accuracy

To make the error < 0.01, need $b_{N+1} < 0.01$, i.e. $\frac{1}{N+1} < 0.01$, so $N + 1 > 100$, $N \geq 100$. Take 100 terms.

Method — The bound is the next term

The rule is simple but strict: the maximum error equals the **size** of the next term. It only applies to alternating series meeting the alternating-test conditions.

Practice 2.1 ▪ 1 mark

State the alternating series error bound.

Solution 2.1

Worked steps

$|S - S_N| \leq b_{N+1}$ (the first omitted term) B1.

Practice 2.2 ▪ 2 marks

For $\sum \frac{(-1)^{n+1}}{n}$ approximated by S_5, bound the error.

Solution 2.2

Method: A worked bound

Worked steps

Error $\leq b_6 = \frac{1}{6} \approx 0.167$ **M1** **A1**.

Practice 2.3 ▪ 2 marks

For $\sum \dfrac{(-1)^n}{n^2}$ approximated by S_2, bound the error.

Solution 2.3

Method: A worked bound

Worked steps

Error $\leq b_3 = \frac{1}{9} \approx 0.111$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

For a convergent alternating series, the error $|S - S_N|$ is at most

A b_N

B b_{N+1}

C S_N

D $\sum b_n$

Solution 3.1

Method: The error bound

A b_N

B b_{N+1} ✓

C S_N

D $\sum b_n$

Worked steps

B — the error is at most the first omitted term b_{N+1}.

Exam-style 3.2 ▪ 1 mark

The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ is approximated by its first three terms.

(a) Write the first omitted term.

(b) Give an upper bound for the error.

Solution 3.2

Method: The error bound

Worked steps

(a) $b_4 = \dfrac{1}{4^3} = \frac{1}{64}$ **B1**. (b) Error $\leq \frac{1}{64} \approx 0.0156$ **M1** **A1**.

Exam-style 3.3 ▪ 3 marks

How many terms of $\sum \frac{(-1)^{n+1}}{n}$ are needed to guarantee an error less than 0.05?

Solution 3.3

Worked steps

Need $b_{N+1} = \frac{1}{N+1} < 0.05$ M1; $N + 1 > 20$, so $N \geq 20$ M1; take 20 terms A1.