Exercise walk-through

Defining Convergent and Divergent Infinite Series ▪ 10.1

eXercise

You must be able to

- define convergence through the limit of **partial sums 部分和**
- compute the first few partial sums
- decide convergence when the partial sums have a known limit

Method — Partial sums

The Nth **partial sum** of $\sum a_n$ is $S_N = a_1 + a_2 + \cdots + a_N$. The series **converges** to L if $S_N \to L$; otherwise it **diverges**.

Method — Computing partial sums

For $\sum \left(\frac{1}{2}\right)^n$ starting at $n = 1$: $S_1 = \frac{1}{2}$, $S_2 = \frac{3}{4}$, $S_3 = \frac{7}{8}$. The partial sums approach 1, so the series converges to 1.

Method — A telescoping series

$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$: $S_N = 1 - \frac{1}{N+1}$, which $\to 1$. Converges to 1.

Method — Divergence by growing sums

For $\sum 1 = 1 + 1 + 1 + \cdots$: $S_N = N \to \infty$, so it diverges. If the partial sums do not settle to a finite value, the series diverges.

Practice 2.1 ▪ 1 mark

Write the third partial sum S_3 of $\sum_{n=1}^{\infty} \frac{1}{n}$.

Solution 2.1

Worked steps

$S_3 = 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$ $\boxed{\textbf{B1}}$.

Practice 2.2 ▪ 1 mark

A series has partial sums $S_N = 3 - \frac{1}{N}$. State its sum.

Solution 2.2

Method: Computing partial sums

Worked steps

Sum $= 3$ (as $N \to \infty$, $\frac{1}{N} \to 0$) **B1**.

Practice 2.3 ▪ 1 mark

A series has $S_N = 2N$. State whether it converges or diverges.

Solution 2.3

Method: Partial sums

Worked steps

Diverges ($S_N \to \infty$) **B1**.

Exam-style 3.1 ▪ 1 mark

A series $\sum a_n$ converges if

A $a_n \to 0$

B the partial sums S_N approach a finite limit

C a_n is always positive

D there are infinitely many terms

Solution 3.1

A $a_n \to 0$

B the partial sums S_N approach a finite limit ✓

C a_n is always positive

D there are infinitely many terms

Worked steps

B — convergence is defined by the partial sums approaching a finite limit.

Exam-style 3.2 ▪ 2 marks

For the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$:

(a) Write S_3.

(b) Find a formula for S_N and hence the sum.

Solution 3.2

Method: A telescoping series

Worked steps

(a) $S_3 = (1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) = 1 - \frac{1}{4} = \frac{3}{4}$ **M1** **A1**. (b) $S_N = 1 - \frac{1}{N+1}$ **M1** **M1**; as $N \to \infty$, sum $= 1$ **A1**.

Exam-style 3.3 ▪ 2 marks

A series has partial sums $S_N = \dfrac{N}{N+1}$. State the sum of the series, with a reason.

Solution 3.3

Worked steps

$\dfrac{N}{N+1} \to 1$, so the series converges to 1 **M1** **A1**.