Exercise walk-through

Determining Limits Using the Squeeze Theorem ▪ 1.8

eXercise

You must be able to

- state the **Squeeze Theorem 夹逼定理**
- apply it when a function is bounded between two others with equal limits

Method — The Squeeze Theorem

If $g(x) \leq f(x) \leq h(x)$ near a and

$$\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L,$$

then $\lim_{x \to a} f(x) = L$.

Method — Example

Since $-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$ and both bounds $\to 0$ at 0, the middle limit is 0.

Practice 2.1 ▪ 1 mark

State the Squeeze Theorem.

Solution 2.1

Method: The Squeeze Theorem

Worked steps

if $g \leq f \leq h$ near a and g, h have the same limit L at a, then f also has limit L B1.

Practice 2.2 ▪ 1 mark

State what must be true of the two bounding limits.

Solution 2.2

Worked steps

they must be equal (both equal L) **B1**.

Practice 2.3 ▪ 2 marks

If $-x^2 \leq f(x) \leq x^2$ and both bounds $\to 0$ at 0, state $\lim_{x\to 0} f(x)$.

Solution 2.3

Method: Example

Worked steps

$\lim_{x\to 0} f(x) = 0$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

The Squeeze Theorem requires $g \leq f \leq h$ and

A $\lim g = \lim h$

B $g = h$ everywhere

C $f = 0$

D $h = 0$

Solution 3.1

Method: The Squeeze Theorem

A $\lim g = \lim h$ ✓

B $g = h$ everywhere

C $f = 0$

D $h = 0$

Worked steps

A.

Exam-style 3.2 ▪ 1 mark

If both bounds approach 3, then $\lim f$ is

- **A** 0
- **B** 3
- **C** 6
- **D** undefined

Solution 3.2

Method: Example

A 0

B 3 ✓

C 6

D undefined

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$-x^2 \leq f(x) \leq x^2$ near 0.
(a) State $\lim_{x\to 0}(-x^2)$.
(b) State $\lim_{x\to 0} x^2$.
(c) State $\lim_{x\to 0} f(x)$.

Solution 3.3

Worked steps

(a) 0 **B1**. (b) 0 **B1**. (c) 0 **B1**.