Exercise walk-through

Determining Limits Using Algebraic Manipulation ▪ 1.6

eXercise

You must be able to

- recognise an **indeterminate form 未定式** $\frac{0}{0}$
- factor and cancel to evaluate the limit

Method — Algebraic manipulation

When direct substitution gives $\frac{0}{0}$, factor and cancel the common factor, then substitute.

Method — Example

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x - 2} = \lim_{x \to 2} (x + 2) = 4.$$

Practice 2.1 ▪ 1 mark

State what to do when substitution gives $\frac{0}{0}$.

Solution 2.1

Method: Algebraic manipulation

Worked steps

factor and cancel the common factor, then substitute **B1**.

Practice 2.2 ▪ 1 mark

Simplify $\dfrac{x^2-9}{x-3}$.

Solution 2.2

Worked steps

$x + 3$ (for $x \neq 3$) B1.

Practice 2.3 ▪ 2 marks

Evaluate $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$.

Solution 2.3

Worked steps

$\lim_{x\to 3}(x+3) = 6$ M1 A1.

Exam-style 3.1 ▪ 1 mark

If direct substitution gives $\frac{0}{0}$, you should

A conclude the limit is 0

B factor and cancel

C stop — no limit exists

D divide by x

Solution 3.1

Method: Algebraic manipulation

A conclude the limit is 0

B factor and cancel ✓

C stop — no limit exists

D divide by x

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

$\dfrac{x^2-1}{x-1}$ simplifies to

A $x-1$

B $x+1$

C x^2+1

D 1

Solution 3.2

A $x - 1$

B $x + 1$ ✓

C $x^2 + 1$

D 1

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$\lim_{x\to 1} \dfrac{x^2-1}{x-1}$.

(a) Factor the numerator.

(b) Cancel $(x-1)$.

(c) Substitute $x=1$.

Solution 3.3

Method: Algebraic manipulation

Worked steps

(a) $(x-1)(x+1)$ **B1**. (b) $x+1$ **B1**. (c) 2 **B1**.