Exercise walk-through

Working with the Intermediate Value Theorem (IVT) ▪ 1.16

eXercise

You must be able to

- state the **Intermediate Value Theorem 介值定理**
- use it to show a value (e.g. a zero) is attained

Method — The IVT

If f is **continuous** on $[a, b]$ and N is any value between $f(a)$ and $f(b)$, then there is at least one c in (a, b) with $f(c) = N$.
A sign change ($f(a)$ and $f(b)$ opposite signs) guarantees a **zero** in between.

Method — Example

If f is continuous with $f(1) = -2$ and $f(3) = 4$, the IVT guarantees a zero in $(1, 3)$.

Practice 2.1 ▪ 1 mark

State the IVT.

Solution 2.1

Worked steps

if f is continuous on $[a, b]$ and N lies between $f(a)$ and $f(b)$, then $f(c) = N$ for some c in (a, b) **B1**.

Practice 2.2 ▪ 1 mark

State the condition f must satisfy for the IVT.

Solution 2.2

Worked steps

it must be continuous on $[a, b]$ **B1**.

Practice 2.3 ▪ 2 marks

If $f(1) = -2$, $f(3) = 4$, and f is continuous, must f have a zero in $(1, 3)$?

Solution 2.3

Worked steps

yes — f is continuous and changes sign, so by the IVT there is a zero in $(1, 3)$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

The IVT requires f to be

- **A** differentiable
- **B** continuous on $[a, b]$
- **C** linear
- **D** a polynomial

Solution 3.1

A differentiable

B continuous on $[a, b]$ ✓

C linear

D a polynomial

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

If $f(2) = -1$ and $f(5) = 3$ (continuous), the IVT guarantees a c with

A $f(c) = 10$

B $f(c) = 0$ for some c in $(2, 5)$

C $f(c) = \infty$

D no such c

Solution 3.2

A $f(c) = 10$

B $f(c) = 0$ for some c in $(2, 5)$ ✓

C $f(c) = \infty$

D no such c

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

f is continuous, $f(0) = -5$, $f(4) = 3$.
(a) State $f(0)$ and $f(4)$.
(b) Note the sign change.
(c) State what the IVT guarantees.

Solution 3.3

Method: The IVT

Worked steps

(a) $f(0) = -5$, $f(4) = 3$ **B1**. (b) it changes from negative to positive **B1**. (c) a zero (a c with $f(c) = 0$) in $(0, 4)$ **B1**.