Past-paper walk-through

Connecting Limits at Infinity and Horizontal Asymptotes ▪ 1.15

eXercise

Questions in this set

- 2025 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(a) Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

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(b) Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \le t \le 4$. Show the setup for your calculations.

Question 1

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An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$. (Note: Your calculator should be in radian mode.)

(c) Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.
(Note: Your calculator should be in radian mode.)

(d) At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

Solution 1

(a)

Mark scheme

- Average value $= \dfrac{1}{4-0}\int_0^4 C(t)\,dt$. Since $\int_0^4 C(t)\,dt = 11.112896$, the average $= \dfrac{1}{4}(11.112896) = 2.778224 \approx 2.778$ acres. (1 pt: correct integral/average-value setup with division by 4; 1 pt: correct answer, accurate to 3 decimal places.)

Solution 1

(b)

Mark scheme

- Average rate of change of C on $[0,4]$: $\dfrac{C(4)-C(0)}{4-0} = 1.282008$. Set $C'(t) = \dfrac{38}{25+t^2}$ equal to this and solve: $\dfrac{38}{25+t^2} = 1.282008 \Rightarrow t = 2.154298 \approx 2.154$. (1 pt: uses/presents the average rate of change; 1 pt: correct answer with supporting work.)

Solution 1

(c)

Mark scheme

- $\lim_{t\to\infty} C'(t) = \lim_{t\to\infty} \dfrac{38}{25+t^2} = 0$. (1 pt: correct limit expression $\lim_{t\to\infty} C'(t)$ or $\lim_{t\to\infty} C(t)$; 1 pt: correct value, 0.)

Solution 1

(d)

Mark scheme

- $A(t) = C(t) - \int_4^t 0.1 \ln x \, dx \Rightarrow A'(t) = C'(t) - 0.1 \ln t$. Setting $A'(t) = 0$: $C'(t) = 0.1 \ln t \Rightarrow t = 11.441700$. Candidates test on $[4, 36]$: $A(4) = 5.128031$, $A(11.4417) = 7.316978$, $A(36) = 1.743056$. The maximum occurs at $t = 11.442$ (or 11.441) weeks. (1 pt: considers $A'(t) = 0$; 1 pt: justification via global/candidates-test argument; 1 pt: correct answer with supporting work.)