Exercise walk-through

Introducing Calculus: Can Change Occur at an Instant? ▪ 1.1

eXercise

You must be able to

- find an **average rate of change** 平均变化率 over an interval
- describe the **instantaneous rate of change** 瞬时变化率 as a limit of average rates

Method — Average vs instantaneous rate

The **average rate of change** of f over $[a, b]$ is $\dfrac{f(b) - f(a)}{b - a}$.

The **instantaneous rate** at a point is the **limit** of these average rates as the interval shrinks to zero — this is the idea calculus makes precise.

Method — Example

$f(x) = x^2$ on $[1, 3]$: average rate $= \dfrac{9 - 1}{3 - 1} = 4.$

Practice 2.1 ▪ 1 mark

State the average rate of change of f on $[a, b]$.

Solution 2.1

Method: Average vs instantaneous rate

Worked steps

$$\frac{f(b)-f(a)}{b-a}$$ B1.

Practice 2.2 ▪ 1 mark

State what the instantaneous rate of change is the limit of.

Solution 2.2

Method: Average vs instantaneous rate

Worked steps

the average rates of change over intervals shrinking to zero B1.

Practice 2.3 ▪ 2 marks

For $f(x) = x^2$, find the average rate of change on $[1, 3]$.

Solution 2.3

Method: Average vs instantaneous rate

Worked steps

$$\frac{f(3)-f(1)}{3-1}=\frac{9-1}{2}=4 \quad \text{M1 A1}.$$

Exam-style 3.1 ▪ 1 mark

The instantaneous rate of change is

A the average over $[a, b]$

B the limit of average rates as the interval shrinks

C always zero

D the slope of a single secant line

Solution 3.1

Method: Average vs instantaneous rate

A the average over $[a, b]$

B the limit of average rates as the interval shrinks ✓

C always zero

D the slope of a single secant line

Worked steps

B.

Exam-style 3.2 ▪ 1 mark

The average rate of change of f on $[a, b]$ is

A $f(b) \cdot f(a)$

B $\dfrac{f(b) - f(a)}{b - a}$

C $f(b) + f(a)$

D $b - a$

Solution 3.2

Method: Average vs instantaneous rate

A $f(b) \cdot f(a)$

B $\dfrac{f(b) - f(a)}{b - a}$ ✓

C $f(b) + f(a)$

D $b - a$

Worked steps

B.

Exam-style 3.3 ▪ 1 mark

$f(x) = x^2$.
(a) Find $f(1)$ and $f(3)$.
(b) Find the average rate of change on $[1, 3]$.
(c) State what a shrinking interval approaches.

Solution 3.3

Method: Average vs instantaneous rate

Worked steps

(a) $f(1) = 1$, $f(3) = 9$ B1. (b) $\dfrac{9-1}{2} = 4$ B1. (c) the instantaneous rate of change B1.