

Learn these by heart before the exam.

## BC-only integration techniques

$\int u dv = uv - \int v du$  choose  $u$  by LIATE: log, inverse trig, algebraic, trig, exponential

$\frac{P(x)}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$  only for NON-repeated linear factors on the BC exam

$\int_a^\infty f(x) dx = \lim_{b \rightarrow \infty} \int_a^b f(x) dx$  write the limit explicitly; the setup earns the mark

$\int_1^\infty \frac{1}{x^p} dx$  converges  $\iff p > 1$  p-

## Differential equations (BC)

$\frac{dy}{dt} = ky \left(1 - \frac{y}{L}\right)$ ,  $y = \frac{L}{1 + Ae^{-kt}}$  fastest growth at  $y = L/2$ ;  $y \rightarrow L$  as  $t \rightarrow \infty$

$y_{n+1} = y_n + h \cdot f(x_n, y_n)$  under-estimates when the solution is concave up

## Parametric, polar, and vectors

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ ,  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) / \frac{dx}{dt}$  the second derivative divides by  $dx/dt$  again, NOT by  $d^2x/dt^2$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$\vec{v}(t) = \langle x'(t), y'(t) \rangle$ , speed =  $|\vec{v}| = \sqrt{x'^2 + y'^2}$

distance =  $\int_a^b |\vec{v}(t)| dt$

$x = r \cos \theta$ ,  $y = r \sin \theta$ ,  $r^2 = x^2 + y^2$

$A = \frac{1}{2} \int_\alpha^\beta r^2 d\theta$  for the area BETWEEN two curves use  $\frac{1}{2} \int (r_{out}^2 - r_{in}^2) d\theta$

## Sequences and series - convergence tests

$\sum_{n=0}^\infty ar^n = \frac{a}{1-r}$ ,  $|r| < 1$

$\sum \frac{1}{n^p}$  converges  $\iff p > 1$   $p = 1$  is the harmonic series: it DIVERGES p-

*n*th-term test: if  $a_n \not\rightarrow 0$  the series diverges. If  $a_n \rightarrow 0$  it proves NOTHING.

Ratio test:  $L = \lim |a_{n+1}/a_n|$ .  $L < 1$  converges,  $L > 1$  diverges,  $L = 1$  inconclusive.

Alternating series: converges if terms decrease in size AND tend to 0. Error is smaller than the first omitted term.

Absolute convergence implies convergence.

Conditional means the series converges but the absolute one does not.

Comparison and limit comparison: compare with a *p*-series or geometric series you already know.

## Power series and Taylor

$f(x) = \sum_{n=0}^\infty \frac{f^{(n)}(a)}{n!} (x-a)^n$  Maclaurin is the case  $a = 0$

$e^x = \sum \frac{x^n}{n!}$ ,  $\sin x = \sum \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ ,  $\cos x =$

$\sum \frac{(-1)^n x^{2n}}{(2n)!}$

$\frac{1}{1-x} = \sum_{n=0}^\infty x^n$ ,  $|x| < 1$  differentiate, integrate or substitute into this to build new series fast

$|R_n(x)| \leq \frac{\max |f^{(n+1)}(z)|}{(n+1)!} |x-a|^{n+1}$

Radius of convergence: apply the ratio test and solve  $L < 1$ . Then TEST BOTH ENDPOINTS separately.

## Exam technique

Name the test before using it: 'by the ratio test' or 'by the alternating series error bound'. Unnamed work loses marks.

For series, show the general term and the limit you took. A bare 'converges' earns nothing.

BC still examines all of AB. Revise limits, FTC, and volumes alongside this sheet.