

Learn these by heart before the exam. 考试前把这些内容背熟。

Limits and Continuity

$\lim_{x \rightarrow c} f(x) = L \iff \lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = L$
极限存在的条件

$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$ 两个特殊极限

Continuous at c needs all three: $f(c)$ exists,
 $\lim_{x \rightarrow c} f(x)$ exists, and they are equal.

Indeterminate $\frac{0}{0}$: factor, rationalise, or use L'Hopital.
Never just substitute and stop.

$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$ only for $\frac{0}{0}$ or $\frac{\infty}{\infty}$; verify the
form FIRST 洛必达法则

IVT: f continuous on $[a, b]$ means it takes every value
between $f(a)$ and $f(b)$. Cite continuity explicitly.

Differentiation

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ 导数定义

$(x^n)' = nx^{n-1}, \quad (e^x)' = e^x, \quad (\ln x)' = \frac{1}{x}, \quad (a^x)' = a^x \ln a$ 基本导数

$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x, \quad (\tan x)' = \sec^2 x$ 三角函数导数

$(uv)' = u'v + uv', \quad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ 乘积与商法则

$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$ 链式法则

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad (\arctan x)' = \frac{1}{1+x^2}$ 反三角函数导数

$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$ 反函数导数

Implicit: differentiate both sides in x , attach $\frac{dy}{dx}$ by the
chain rule to every y , then solve.

Contextual Applications

Motion: $v(t) = x'(t), a(t) = v'(t)$. Speed is $|v|$; speed
rises when v and a share a sign.

total distance = $\int_a^b |v(t)| dt$, displacement = $\int_a^b v(t) dt$ 路程与位移

Related rates: write the relation, differentiate in t ,
THEN substitute the instant's values. Never substitute
first.

$L(x) = f(a) + f'(a)(x-a)$ over-estimates when the
curve is concave down 线性近似

Analytical Applications

$f' > 0$ increasing, $f' < 0$ decreasing. $f'' > 0$ concave
up, $f'' < 0$ concave down.

Critical point: $f' = 0$ or undefined. It is a
MAXIMUM if f' changes $+$ to $-$; say that in words.

Inflection point: f'' CHANGES SIGN. $f'' = 0$ alone is
not enough.

$f'(c) = \frac{f(b) - f(a)}{b - a}$ needs continuous on $[a, b]$ and
differentiable on (a, b) ; state both 中值定理

Candidates test for a closed interval: check every
critical point AND both endpoints.

Integration and Accumulation

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1), \quad \int \frac{1}{x} dx = \ln |x| + C$ 基本积分

$\int e^x dx = e^x + C, \quad \int \sin x dx = -\cos x + C, \quad \int \sec^2 x dx = \tan x + C$ 常用积分

$\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x)) g'(x)$ 微积分基本定理 (第一部分)

$\int_a^b f'(x) dx = f(b) - f(a)$ 微积分基本定理 (第二部分)

$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$ 函数平均值

u -substitution: change the LIMITS too, or convert
back to x before evaluating. Do not mix.

Riemann sums: left under-estimates an increasing
function, right over-estimates it. Trapezoid
over-estimates when concave up.

Differential Equations

Separable: get all y with dy and all x with dx ,
integrate both sides, add ONE $+C$, then use the initial
condition.

$\frac{dy}{dt} = ky \Rightarrow y = y_0 e^{kt}$ 指数增长与衰减

Slope field: at each point the segment has slope $\frac{dy}{dx}$.
Follow the segments to sketch a solution curve.

Applications of Integration

$A = \int_a^b [f(x) - g(x)] dx$ top minus bottom; split where
they cross 两曲线间的面积

$V = \pi \int_a^b [R(x)^2 - r(x)^2] dx$ 圆盘/圆环法体积

$V = \int_a^b A(x) dx$ square cross-section $A = s^2$; semicircle
 $A = \frac{\pi}{2} r^2$ 已知截面的体积

Accumulation: $f(b) = f(a) + \int_a^b f'(x) dx$. Read this as
'start plus the change'.

Exam technique

Justify in words, with the theorem named: 'by the MVT, since f is continuous on $[a, b]$ and differentiable on (a, b) '.

Calculator sections: show the SETUP integral or equation. A bare number earns almost nothing.

Keep 3 decimal places and units. $\frac{dy}{dx}$ answers usually need units of y per unit of x .