AP Calculus AB

Formula Sheet · 公式表

AP Calculus supplies no formula sheet of any kind. Every derivative, every antiderivative and every theorem statement has to come from memory, and the free-response is written on the assumption that they do. This is that list, with the conditions each theorem needs – because an invoked theorem whose hypotheses are unstated earns no point. 本科目不提供公式表，以下全部需要背诵，并须能陈述定理条件。

Derivatives you must know cold

Powers, exponentials, logarithms 幂、指数、对数 — $\frac{d}{dx}x^n = nx^{n-1}$; $\frac{d}{dx}e^x = e^x$; $\frac{d}{dx}a^x = a^x \ln a$; $\frac{d}{dx}\ln x = \frac{1}{x}$

Trigonometric 三角函数 — $\sin \to \cos$, $\cos \to -\sin$, $\tan \to \sec^2$, $\sec \to \sec\tan$, $\cot \to -\csc^2$, $\csc \to -\csc\cot$

Inverse trigonometric 反三角函数 — $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$; $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$

The three rules 三大法则 — Product $(uv)' = u'v + uv'$; quotient $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$; chain $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Implicit differentiation 隐函数求导 — Differentiate both sides with respect to x, attaching $\frac{dy}{dx}$ whenever you differentiate a y, then solve for $\frac{dy}{dx}$.

Antiderivatives you must know cold

The basic set 基本积分 — $\int x^n dx = \frac{x^{n+1}}{n+1}$ for $n \neq -1$; $\int \frac{1}{x}dx = \ln|x|$; $\int e^x dx = e^x$; $\int \sin x\,dx = -\cos x$; $\int \sec^2 x\,dx = \tan x$

u-substitution 换元积分 — Choose u as the inside function, compute du, and change the LIMITS as well when the integral is definite. Changing limits and then also substituting back is a contradiction that costs a point.

The absolute value 绝对值 — $\int \frac{1}{x}dx = \ln|x| + C$. Omitting the bars is scored as an error in free-response.

The constant 积分常数 — Every indefinite integral needs $+C$, and a differential-equation solution needs it BEFORE the initial condition is applied, not after.

Theorems – and the hypotheses that must be stated

Intermediate Value Theorem 介值定理 — If f is CONTINUOUS on $[a, b]$ and k lies between $f(a)$ and $f(b)$, some c in (a, b) has $f(c) = k$. Say the word continuous and quote the two endpoint values.

Mean Value Theorem 中值定理 — If f is continuous on $[a, b]$ and DIFFERENTIABLE on (a, b), some c has $f'(c) = \frac{f(b)-f(a)}{b-a}$. Both hypotheses must be written; either one alone is insufficient.

Extreme Value Theorem 最值定理 — A continuous function on a CLOSED interval attains a maximum and a minimum. The closedness is what the exam tests.

Fundamental Theorem, part 1 微积分基本定理一 — $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$, and with an upper limit $g(x)$ the chain rule adds a factor $g'(x)$. Accumulation-function questions are entirely this.

Fundamental Theorem, part 2 微积分基本定理二 — $\int_a^b f'(x)\,dx = f(b) - f(a)$, which is why "the integral of a rate gives the net change" – the sentence the exam wants in words.

Applications and their exact phrasing

Motion 运动 — Velocity is the derivative of position, acceleration of velocity. DISPLACEMENT is $\int v\,dt$; DISTANCE travelled is $\int |v|\,dt$. Speed increases when velocity and acceleration share a sign.

Related rates 相关变化率 — Write the relationship between the quantities, differentiate with respect to time, and substitute the instantaneous values only AFTER differentiating. Substituting first is the standard error and it is unrecoverable.

Riemann sums 黎曼和 — Left, right, midpoint and trapezoidal, from a table of values. Say whether the estimate is an over- or under-estimate and justify it from whether the function is increasing or concave.

Area and volume 面积与体积 — Area between curves $= \int(\text{top}-\text{bottom})\,dx$. Volume by discs $= \pi\int R^2 dx$; washers $= \pi\int(R^2 - r^2)dx$; known cross-sections $= \int A(x)\,dx$. Set up the integral even when the calculator evaluates it – the setup is where the points are.

Separable differential equations 可分离变量方程 — Separate, integrate both sides, add C, then apply the initial condition to find C, then solve for y. Exponential growth is the special case $\frac{dy}{dt} = ky$, giving $y = y_0 e^{kt}$.

How the free-response is graded

On calculator-active parts, write the integral or the equation being solved before giving the numerical answer. A bare number from the calculator earns the answer point only, and loses the setup point.

Answers to applied questions need units and a sentence: “the volume of water is increasing at 3.4 gallons per minute at time $t = 5$”. A bare number is routinely scored zero on interpretation parts.

Justify every extremum and every point of inflection by referring to a sign CHANGE of the appropriate derivative, not merely to it being zero. State where the sign changes and in which direction.