

*Calculus free-response points are lost to language more than to algebra: an answer that computes correctly but never says “rate of change of ... with respect to ...”, or states a theorem without checking its hypotheses. Learn these sentences and use them verbatim. 微积分失分多在表述: 必须写清 “关于 ..... 的变化率”, 并核对定理成立条件。*

## Limits and continuity

**Limit 极限** — the value a function approaches as the input approaches a point; it can exist where the function is undefined

**Continuity at a point 在一点连续** —  $f$  is continuous at  $c$  when  $f(c)$  exists, the limit as  $x \rightarrow c$  exists, and the two are equal – all three conditions

**Intermediate Value Theorem 介值定理** — if  $f$  is continuous on  $[a, b]$  and  $k$  lies between  $f(a)$  and  $f(b)$ , then  $f(x) = k$  for some  $x$  in  $(a, b)$ ; continuity must be stated to use it

**Removable discontinuity 可去间断点** — a point where the limit exists but does not equal the function value, so redefining one point would make  $f$  continuous

## Derivatives

**Derivative 导数** — the instantaneous rate of change of  $f$  with respect to  $x$ , defined as the limit of the difference quotient

**Differentiability 可导性** — the existence of the derivative at a point; differentiability implies continuity, but continuity does not imply differentiability

**Chain rule 链式法则** — the derivative of a composite function is the derivative of the outer function evaluated at the inner, times the derivative of the inner

**Implicit differentiation 隐函数求导** — differentiating both sides of a relation with respect to  $x$ , treating  $y$  as a function of  $x$  so that  $dy/dx$  appears by the chain rule

**Critical point 临界点** — a point in the domain where the derivative is zero or undefined

**Mean Value Theorem 中值定理** — if  $f$  is continuous on  $[a, b]$  and differentiable on  $(a, b)$ , some  $c$  in  $(a, b)$  has  $f'(c)$  equal to the average rate of change over  $[a, b]$

**Point of inflection 拐点** — a point where the concavity changes, which requires the second derivative to CHANGE SIGN, not merely to be zero

**Related rates 相关变化率** — using a relation between quantities and the chain rule to find one rate from another, both with respect to time

## Integrals

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**Definite integral** 定积分 — the limit of Riemann sums over a partition, equal to the net signed area between the curve and the x-axis

**Riemann sum** 黎曼和 — an approximation of a definite integral by rectangles; left, right, midpoint and trapezoidal rules differ in the height chosen

**Fundamental Theorem of Calculus** 微积分基本定理 — differentiation and integration are inverse: the derivative of the accumulation function is the integrand, and a definite integral equals the change in an antiderivative

**Average value of a function** 函数平均值 — the definite integral over an interval divided by the LENGTH of that interval

**Accumulation function** 累积函数 — a function defined as an integral from a fixed lower limit to a variable upper limit; its derivative is the integrand

**Separable differential equation** 可分离变量微分方程 — an equation that can be written so all y terms are on one side and all x terms on the other, then integrated

**Slope field** 斜率场 — a grid of short segments whose slopes are given by  $dy/dx$  at each point, showing the shape of the solution curves

## Interpretation in context

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Every rate answer needs UNITS and a reference: “the volume is increasing at  $3 \text{ cm}^3$  per second at  $t = 4$ ”, not “3”.

A definite integral of a rate gives the NET CHANGE in the quantity, and saying so in context is usually a separate point.

Before invoking the Mean Value Theorem or the Intermediate Value Theorem, state that the hypotheses hold. The justification is the mark, not the conclusion.