7 Solving simple differential equations – Part 2

Name: ____________ Class: ________ Date: __________

Recall

In Part A you met differential equations. Now solve simple ones:

- Solving means finding the function from its rate of change.
- One special kind describes things that grow by a fixed percentage – like money or populations.

Each idea has a worked example before the Practice.

New ideas

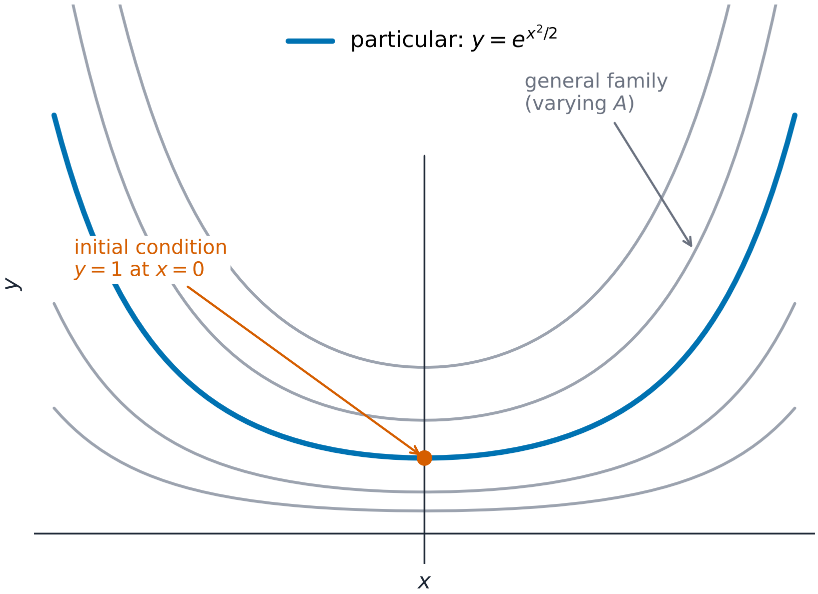
1 Solving by integrating

If $\dfrac{dy}{dx}$ is given, integrate to find y.

Worked example. $\dfrac{dy}{dx} = 3x^2$ integrates to $y = x^3 + C$.

2 A family of curves

The constant C gives a **family** of solution curves; one extra fact (a known point) picks out the right one.



3 Exponential growth

A very common equation is $\frac{dy}{dt} = ky$ – "the rate of growth is proportional to how much there is". Its solution is exponential growth, $y = Ce^{kt}$.

4 Where it appears

This models populations, money earning interest, and radioactive decay (with a negative k) – anything that grows or shrinks by a fixed proportion.

Watch out: exponential growth ($\frac{dy}{dt} = ky$) means the quantity grows **faster and faster** as it gets bigger, because the rate depends on the current amount. It is not steady, straight-line growth.

Practice

Try these in order. The methods are all above.

2.1 State how you solve $\dfrac{dy}{dx} = f(x)$ for y. [1]

2.2 Solve $\dfrac{dy}{dx} = 2x$. [2]

2.3 Solve $\dfrac{dy}{dx} = 4x^3$. [2]

2.4 State the type of growth described by $\dfrac{dy}{dt} = ky$. [1]

2.5 Give two real situations modelled by exponential growth or decay. [2]