

7 Differential equations and slope fields

– Part 1

Name: _____ Class: _____ Date: _____

Recall

You have solved ordinary equations to find a number:

- An equation like $2x + 1 = 7$ has the solution $x = 3$.
- But some equations describe how a quantity **changes** – they involve a derivative.

This sheet introduces differential equations. Each idea has a worked example.

New ideas

■ 1 What a differential equation is

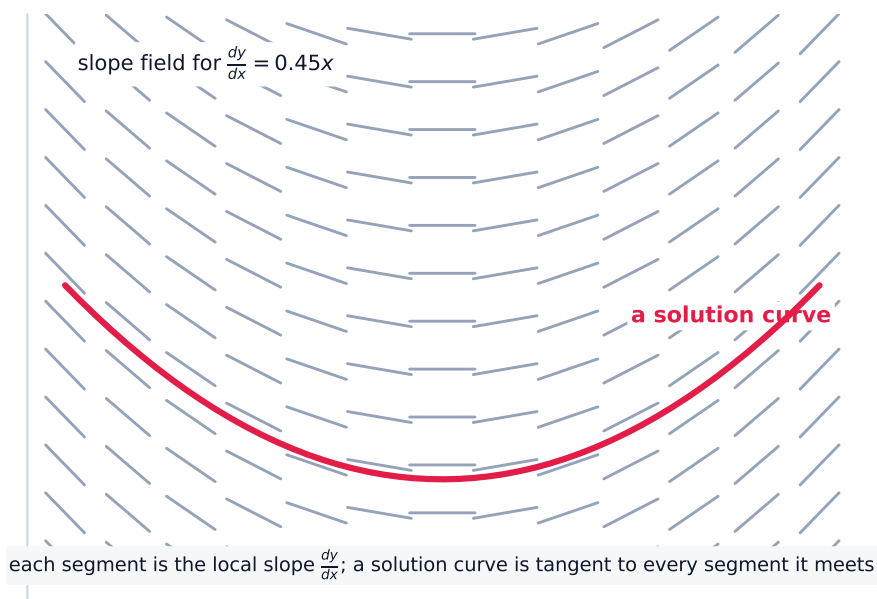
A **differential equation** involves a **derivative** – it tells you the **rate of change**, not the value directly. For example $\frac{dy}{dx} = 2x$ says "the slope at each point is $2x$ ".

■ 2 Its solution is a function

Solving it means finding the **function** y whose derivative matches. For $\frac{dy}{dx} = 2x$, one solution is $y = x^2$ (its derivative is $2x$).

■ 3 Slope fields

A **slope field** draws a tiny line at many points showing the slope there. It lets you see the shape of the solutions without solving.



■ 4 A family of solutions

There are usually **many** solutions differing by a constant (like $y = x^2$, $y = x^2 + 1$, ...)
– a whole family that follows the slope field.

Watch out: a differential equation gives the **slope** (dy/dx), not the function itself.
You must integrate (or otherwise solve) to recover the actual function y .

Practice

Try these in order. The methods are all above.

2.1 State what makes an equation a “differential equation”. [2]

2.2 State what it means to solve a differential equation. [2]

2.3 For $\frac{dy}{dx} = 2x$, verify that $y = x^2$ is a solution. [2]

2.4 State what a slope field shows. [2]

2.5 State why a differential equation usually has many solutions. [1]
