6 Integration and the power rule – Part 2

Name: ____________ Class: ________ Date: __________

Recall

In Part A you saw that the integral is the exact area under a curve. Now compute it:

- Integration is the **reverse** of differentiation.
- There is a simple rule for powers of x, just like the power rule for derivatives.

Each idea has a worked example before the Practice.

New ideas

1 Integration undoes differentiation

Integration finds a function whose derivative is the one you started with – it is the reverse of differentiating. This reverse is written with the sign $\int$.

2 The power rule for integration

To integrate a power of x, **raise the power by one and divide by the new power**:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C.$$

Worked example. $\int x^2 \, dx = \dfrac{x^3}{3} + C$ (check: differentiating $\frac{x^3}{3}$ gives x^2).

3 The "+ C"

Because many functions share the same derivative (they differ by a constant), we add a constant C when integrating.

4 Definite integral gives area

A **definite integral** (with limits) gives the exact area under the curve between two x-values.

Worked example. The area under $y = x$ from 0 to 2 is $\left[\frac{x^2}{2}\right]_0^2 = \frac{4}{2} - 0 = 2$.

Watch out: integration **raises** the power and **divides** – the exact opposite of differentiation, which lowers the power and multiplies. Mixing them up is a common slip.

Practice

Try these in order. The methods are all above.

2.1 State how integration relates to differentiation. [1]

2.2 Find $\int x^3 \, dx$. [2]

2.3 Find $\int x^4 \, dx$. [2]

2.4 State why we add "$+C$" when integrating. [2]

2.5 Find the area under $y = x$ from 0 to 4 using $\left[\frac{x^2}{2}\right]_0^4$. [3]