

8.1 Average Value of a Function

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS average value 平均值

Objective

Build the skills to answer exam questions on the **average value of a function** — the integral divided by the width.

You must be able to:

- apply $f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx$
- distinguish the **average value** 平均值 of a function from the average **rate of change**
- interpret the result in context

1 Worked examples

Study these first. Each one shows the method for a question type used later — follow the steps and you can do the Practice and Exam-style questions yourself.

■ The average-value formula

The average value of f over $[a, b]$ is

$$f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) dx.$$

It is the constant height of a rectangle with the same area as the region under f .

■ A worked value

Average of $f(x) = x^2$ on $[0, 3]$: $\frac{1}{3-0} \int_0^3 x^2 dx = \frac{1}{3} \left[\frac{x^3}{3} \right]_0^3 = \frac{1}{3} \cdot 9 = 3.$

■ Average value vs average rate of change

- **Average value** of f uses an **integral**: $\frac{1}{b-a} \int_a^b f.$
- **Average rate of change** of f uses a **difference**: $\frac{f(b) - f(a)}{b-a}.$

Read the question carefully to pick the right one.

■ **In context**

If $v(t)$ is speed, its average value $\frac{1}{b-a} \int_a^b v \, dt$ is the **average speed** over the interval — total distance divided by time.

2 Practice

Now apply the methods above.

2.1 Write the formula for the average value of f on $[2, 6]$. [1]

2.2 Find the average value of $f(x) = 4x$ on $[0, 5]$. [3]

2.3 Find the average value of $f(x) = 6x^2$ on $[0, 2]$. [3]

3 Exam-style questions

3.1 The average value of f on $[a, b]$ is [1]

- **A** $f(b) - f(a)$
 - **B** $\frac{f(b) - f(a)}{b - a}$
 - **C** $\frac{1}{b - a} \int_a^b f \, dx$
 - **D** $\int_a^b f \, dx$
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3.2 A tank's inflow rate is $r(t) = 12 - t$ litres/min for $0 \leq t \leq 8$.

(a) Find the average inflow rate over this interval. [3]

(b) State the units of your answer.

[1]

3.3 The temperature over a day is modelled by $T(t) = 20 + 6t - t^2$ for $0 \leq t \leq 6$ hours. Find the average temperature over the interval.

[4]

Solutions

$$\mathbf{2.1} \quad \frac{1}{6-2} \int_2^6 f(x) \, dx.$$

$$\mathbf{2.2} \quad \frac{1}{5} \int_0^5 4x \, dx = \frac{1}{5} [2x^2]_0^5 = \frac{1}{5}(50) = 10.$$

$$\mathbf{2.3} \quad \frac{1}{2} \int_0^2 6x^2 \, dx = \frac{1}{2} [2x^3]_0^2 = \frac{1}{2}(16) = 8.$$

3.1 C — the average value is the integral divided by the width.

$$\mathbf{3.2} \quad (\text{a}) \quad \frac{1}{8} \int_0^8 (12 - t) \, dt = \frac{1}{8} [12t - \tfrac{t^2}{2}]_0^8 = \frac{1}{8}(96 - 32) = 8. \quad (\text{b}) \text{ litres per minute.}$$

$$\mathbf{3.3} \quad \frac{1}{6} \int_0^6 (20 + 6t - t^2) \, dt = \frac{1}{6} [20t + 3t^2 - \tfrac{t^3}{3}]_0^6 = \frac{1}{6}(120 + 108 - 72) = \frac{1}{6}(156) = 26^\circ.$$